

Developing computer-assisted learning tool based on discovery learning to improve students' mathematical reasoning

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Abstrak Kemampuan penalaran matematika dibutuhkan dalam langkah-langkah penyelesaian dengan matematika secara numerik. Mahasiswa pendidikan matematika selalu kesulitan dalam membangun langkah penyelesaian menjadi algoritma, dan tidak mampu membangun kode program. Penyebab kesulitan dan kegagalan tersebut adalah hambatan belajar berupa *didactical obstacle*. Pembelajaran tidak memfasilitasi pengembangan kemampuan penalaran matematis dalam membangun algoritma dan kode program. Bukan hanya penggunaan metode ajar yang tidak sesuai, tetapi juga keterbatasan alat-alat ajar, seperti komputer dalam pembuatan sebuah program. Masalah ini dihadapi oleh mahasiswa matematika dan informatika dalam proses belajar pada matakuliah metode numerik, yaitu rendahnya kemampuan penalaran matematis serta keterbatasan penggunaan fasilitas lab komputer di jurusannya. Penelitian yang dilakukan adalah membangun dan mengevaluasi pembelajaran berbantuan komputer yang berbasis *discovery learning*. Metode dalam penelitian ini menggunakan pendekatan kuantitatif dalam memperoleh evaluasi secara kuantitas dari pengembangan pembelajaran berbantuan komputer menggunakan web yang dikembangkan melalui *research and development* dengan model ADDIE. Web yang digunakan sebagai alat pembelajaran berbantuan komputer dikategorikan sangat valid oleh ahli media, ahli matematika, dan uji responden. Hasil pre-tes dan post-tes kemampuan penalaran matematis dianalisis dengan uji paired t-test memperoleh nilai signifikan sebesar 0.609 diatas level signifikansi 0.05 yang menunjukkan bahwa pembelajaran berbantuan komputer melalui web berbasis *discovery learning* dapat digunakan untuk meningkatkan kemampuan penalaran matematis mahasiswa. Observasi langsung juga memperlihatkan peningkatan keterlibatan dan memfasilitasi peningkatan kemampuan penalaran matematis mahasiswa. Kelemahan yang harus diperhatikan adalah penggunaan waktu yang relatif lebih lama karena pengajar memperoleh banyak tanggapan dari mahasiswa serta memberikan arahan berulang kali sehingga sesuai dengan langkah *discovery learning* di dalam media web.

Kata kunci Pembelajaran berbantuan komputer, Media web, Pembelajaran discovery, Penalaran matematis

Abstract Mathematical reasoning abilities were needed in solving steps using mathematics numerically. In mathematics education, students always had difficulty building solution steps into algorithms and were unable to build program code. The cause of these difficulties and failures was learning obstacles in the form of didactical obstacles. Learning did not facilitate the development of mathematical reasoning abilities for building algorithms and program code. It was not only the use of inappropriate teaching methods but also the limitations of teaching tools, such as computers, in creating a program. Mathematics and informatics students encountered this issue during their learning journey in numerical methods courses, experiencing both low mathematical reasoning abilities and restricted access to computer lab facilities within their department. The research carried out was to develop and evaluate computer-assisted learning based on discovery learning. The method in this research used a quantitative approach to obtain a quantitative evaluation of the development of computer-assisted learning using the web, which was developed through research and development in the ADDIE model. The web was used as a computer-assisted learning tool in the

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very valid category by media experts, mathematicians, and test respondents. The results of pre- and post-tests of mathematical reasoning ability were analyzed using a paired t-test, obtaining a significant value of 0.609 above the significance level of 0.05, indicating that computer-assisted learning through web-based discovery learning could be used to enhance students' mathematical reasoning abilities. Direct observations also showed increased engagement and facilitated improvement in students' mathematical reasoning abilities. A weakness to note was the relatively longer time usage, as instructors received many responses from students and provided repeated guidance to align with the steps of discovery learning within the web media.

Keywords *Computer-assisted learning, Web media, Discovery learning, Mathematical reasoning*

Introduction

Reasoning abilities in general are an important part of high-level thinking abilities, namely the abilities needed to face everyday life in complex real-world situations. Without reasoning, the knowledge and experience possessed will be useless because it cannot be applied to new situations (Bhat, 2014). Likewise, mathematical reasoning abilities have an important role as students' thinking power or ability to solve mathematical problems. The importance of mathematical reasoning abilities has become a global concern, one of which is that PISA adjusted the assessment of reasoning abilities to be increased by 25% (Bernhardt et al., 2014; Nikou & Aavakare, 2021, Ma & Qin, 2021). This phenomenon shows mathematical reasoning ability as a core ability that must be possessed by the international community in the 21st century.

In the scope of education, mathematical reasoning abilities increase learning success. Several studies show that mathematical reasoning abilities or logistical reasoning are related to the level of academic achievement in mathematics, while reasoning abilities are implicitly visible in literacy skills based on academic achievement (Bayazit, 2013; Hidayat et al., 2018). For the case is solving mathematical problems, solving problems numerically requires reasoning abilities to be able to develop solution steps, starting from formal mathematical modeling and formulation. To achieve this, mathematical reasoning skills are required, which include the ability to connect mathematical problems with numerical solutions, apply systematic methods until gets a solution, and evaluate its correctness (Bernard & Chotimah, 2018). On the other hand, learning experience in solving using numerical methods can be used to train mathematical reasoning abilities.

Skemp in Hartati (2016), to learn conceptual and metacognitive knowledge mathematical abilities are needed, one of which is mathematical reasoning so that mathematical reasoning abilities are the basic knowledge that must be possessed to learn computational algorithms. This failure is closely related to weak mathematical reasoning abilities, showed by initial observations of mathematics and informatics students were not able to develop solution steps using numerical methods, starting from modeling to building programs. Most students fail in building formulations and developing them into algorithms. The findings from a meta-analysis exploring the notable connection between reasoning skills and algorithm development suggest that a deficiency in reasoning abilities can result in challenges when tackling programming problems (Jones D., 2020). This problem is the main reason for efforts to improve reasoning abilities for students in numerical methods subjects. This indication serves as the primary rationale for initiating new research, which is to create a website as a digital learning platform based on

discovery learning for students in the numerical methods course to enhance their mathematical reasoning skills.

Increasing mathematical reasoning abilities is sought through the learning process, namely applying appropriate learning models and supporting learning media. Several previous studies revealed that the application of the Discovery learning model was able to improve mathematical reasoning abilities (Anggraini, 2018; Putri, D.A, et al., 2017; Yuliana N, 2018). In addition, the support of intelligent digital media (computers) increases the possibility of successful learning (Mayer, R. E., 2014; Murillo-Zamorano et al., 2019; Xiao and Sun, 2021). The application of computer-assisted Discovery learning is building a learning web as a medium while internalizing the Discovery learning steps. Furthermore, the results of web media development are applied in numerical methods classes and assess the increase in students' reasoning abilities.

Theoretical review

Computer assisted learning

Web-based learning can be considered as part of Computer Assisted Learning (CAL) because it uses computer technology as a tool in the learning process. Bates, A. W. (2015). The following is an explanation of how Web-based learning can be considered as part of CAL (Bates et al., 2015; Mayer et al., 2014; Clark et al., 2016):

- a. Use of Computer Technology: Web-based learning relies on computer technology, especially the internet and the World Wide Web (WWW), to present learning material to users. Thus, every learning interaction that occurs via the web is accessed using a computer device, such as a personal computer, laptop, or mobile device.
- b. Interactive and Multimedia-Based: Web-based learning offers an interactive and multimedia-based learning experience. Through the web, students can access various types of learning materials, including text, images, audio, video, animation and interactive. This allows for more engaging, in-depth and comprehensive learning.
- c. Access from Anywhere and Anytime: One of the main advantages of web-based learning is its flexibility. Students can access learning materials, assignments, quizzes, and other resources from anywhere and at any time, as long as they have an internet connection. This allows for independent learning and is tailored to individual schedules and preferences.
- d. Collaboration and Interaction: Web-based learning facilitates collaboration and interaction between students and teachers through various online features, such as discussion forums, chat rooms, video conferencing, and other collaboration tools. This allows students to share thoughts, experiences, and ideas with fellow students and educators, creating a collaborative and supportive learning environment.

Thus, Web-based learning can be considered a form of Computer Assisted Learning (CAL) because it uses computer technology as the main tool in presenting learning material, facilitating interaction between students and teachers, and creating an interactive and flexible learning experience.

Discovery-based computer assisted learning

Discovery-based computer assisted learning is a learning approach that allows students to be actively involved in the process of discovering or exploring learning material using the help

of computer technology (Kirschner et al., 2016). According to Budiningsih (2005), the Discovery Learning Model is a way of learning to understand concepts, meanings and relationships through an intuitive process to finally arrive at a conclusion. According to Syah (2004) in applying the Discovery Learning method in the classroom, there are several procedures that must be implemented in general teaching and learning activities as shown in Figure 1.

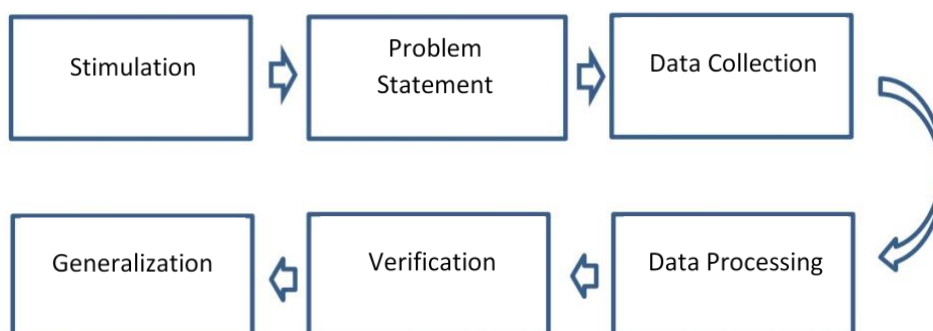


Figure 1. Flow of Discovery Learning steps (Syah, 2004)

Learning that applies computer assisted discovery learning steps can help mathematics learning, especially algebra material (Aftiani et al., 2021). The characteristics of discovery-based computer assisted learning are (Clark & Mayer, 2016; Kirschner et al., 2016):

- a. Student Activities: Students are actively involved in the learning process, which allows them to develop a deeper understanding of the learning material.
- b. Discovery-Based Approach: Students are given the opportunity to discover or explore learning concepts or principles through interaction with the software, simulations or learning applications provided.
- c. Use of Computer Technology: Computer technology is used as a tool to support the student discovery process. This could be simulation software, interactive applications, or other digital learning resources.
- d. Active Involvement: Students are invited to think critically, ask questions, and carry out trials or experiments to gain a better understanding of the learning material.
- e. Collaboration and Discussion: This approach can facilitate collaboration between students, where they can share their own ideas, experiences and discoveries in the learning process.

The benefits of this approach include increasing student motivation and engagement in learning, facilitating deeper understanding of concepts, and developing critical thinking and discovery skills. However, it should be noted that this approach also requires support and guidance from the teacher or facilitator to ensure that students gain the right understanding and in accordance with the learning objectives.

Reasoning abilities in numerical method course

Bjuland & Mosvold (2015) revealed that the complexity of reasoning is related to the process of five mathematical processes; namely: sense-making, conjecturing, convincing, reflecting, and generalizing. Sense-making is the ability to construct a schema for problems and represent knowledge. Sense-making is the process of adapting and associating new information acquired with prior knowledge. This process occurs when the mathematical situation is

understood and then attempts are made to communicate it in mathematical symbols or languages. Conjecturing involves the ability to predict theoretical activities and conclusions based on incomplete facts. Products from conjecturing are achieved through the completion strategy of arguing and communicating mathematical activities. This completion strategy is then implemented through an activity called convincing. Meanwhile, reflecting activities undertaken to re-evaluate the three previous processes. These five abilities are essentially a series of capability processes related to the quality of mathematical reasoning.

Reasoning abilities to engage with numbers, calculations, patterns, and logical and scientific thinking is referred to as logical-mathematical intelligence (Lestari, 2019). This ability is applied in 6 steps in solving mathematics using numerical methods, namely; (1) understanding the problem, (2) selecting methods, (3) forming numerical models, (4) building algorithms, (5) calculations and iteration, and (6) analysis and interpretation (Chapra & Canale, 2014). Mathematical reasoning is an important aspect in the development and application of numerical methods. The following are several mathematical reasoning concepts that are relevant to numerical methods (Chapra & Canale, 2014; Cheney & Kincaid, 2013):

- a. **Understanding Basic Theory:** Mathematical reasoning is required to understand the theoretical basis of numerical methods, including the underlying mathematical principles. This involves understanding concepts such as limits, differentiation, integration, and linear algebra, which are the basis for many numerical methods.
- b. **Error Analysis:** Mathematical reasoning is used to analyze errors that occur in numerical approximations and calculations. This involves estimating the error rate in numerical results, as well as understanding the factors that influence the accuracy of the solution.
- c. **Numerical Stability:** Mathematical reasoning is used to identify the numerical stability of a method, namely the extent to which the method can provide consistent and guaranteed results in different situations, including when rounding errors or other numerical disturbances occur.
- d. **Convergence:** Mathematical reasoning is used to prove the convergence of a numerical method, namely that the resulting numerical solution approaches the true solution as the number of iterations increases. This involves using the convergence theorem from mathematical analysis
- e. **Algorithmic Complexity:** Mathematical reasoning is required to analyze the algorithmic complexity of a numerical method, including the number of mathematical operations required, numerical stability, and memory space required.
- f. **Method Selection:** Mathematical reasoning helps in selecting the most suitable numerical method to solve a particular problem. This involves consideration of the mathematical characteristics of the problem, as well as the advantages and disadvantages of the various numerical methods available.

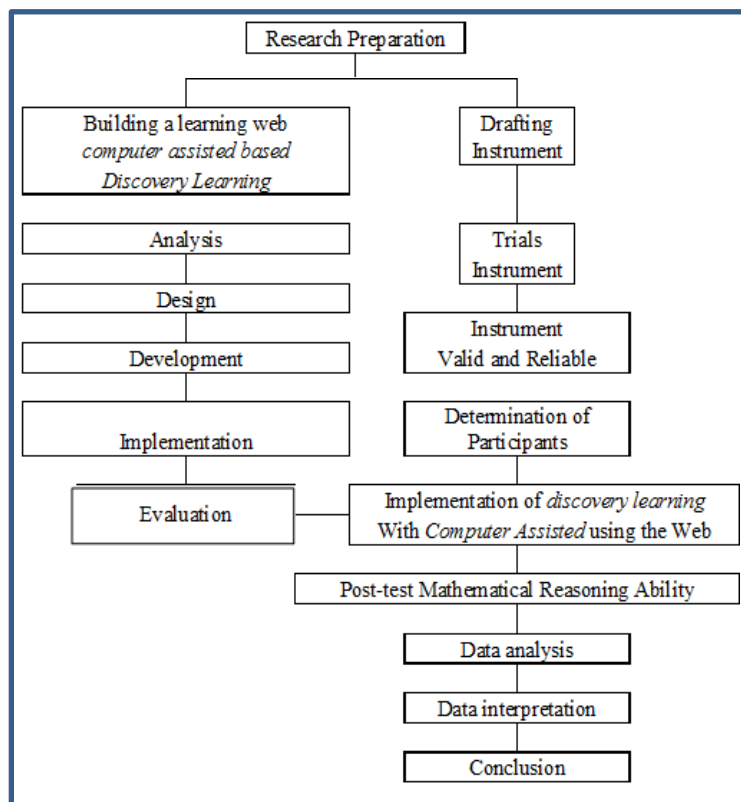


Figure 2. Research flow diagram

Methods

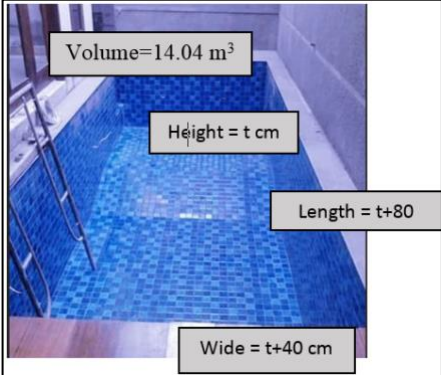
The study was conducted by applying the research and development (R&D) method using the ADDIE steps, and then the results of the web media were implemented as an effort to train mathematical reasoning abilities. Research flow show in [Figure 2](#), there were two research steps conducted: (1) the development of a website as a computer-assisted learning, and (2) statistical analysis of mathematical reasoning abilities. The research subjects were students taking numerical methods courses at a university in Langsa City. The sample selection took into account the conditions of the teaching classes that had been established by the educational institution or unit, so there was no need to regroup random samples (Sugiyono, 2009; Cohen et al., 2007). To see mathematical reasoning abilities, a pretest-posttest Control Group Design was used. In this design, two types of tests are given, namely a pre-test which is carried out before the treatment to see the initial situation, and a post-test which is carried out after the treatment (Creswell, 2010).

Computer assisted web learning based on Discovery learning applies a research and development model using the ADDIE method. Building a web begins by carrying out (1) Analysis which aims to identify problems that cause gaps in the learning process, (2) Design stage is intended to verify the expected product design and formulate testing methods that are in accordance with the preparation of task lists, preparation of objectives performance, and preparing testing strategies by designing and preparing validation sheets, test sheets, (3) The development phase aims to develop starting with content creation with validation by content experts, learning experts and field practitioner experts, (4) Implementation phase is carried out using the web as discovery learning with computer assisted learning, and (5) The evaluation

stage aims to determine the weaknesses of the discovery learning web with computer assisted learning as a basis for improvement. The research instruments used in the assessment and as a data source are mathematical reasoning ability pre-test and post-test. The mathematical reasoning ability is tested based on five indicators of reasoning, namely memory, algorithmic, plausible, novelty, and mathematics foundation reasoning (Lithner, 2012). The test was given to students presented in Table 1.

Table 1. Test instrument for mathematical reasoning

| No | Problem | Sub-question | Expectation of reasoning abilities |
|----|--|---|--|
| 1 | A non-linear equation $x^2 - 3x - 5 = 0$ Determine solutions to the equation using two approaches: analytical method and numerical method, specifically employing the bisection method with the selection of bounds that contain the solution. | <ul style="list-style-type: none"> - Using analytical method - Find the bounds and prove it that contain a solution - Using bisection method to find a solution - Find other solution | <ul style="list-style-type: none"> - Memorize reasoning - Plausible reasoning - Algorithmic reasoning - Novelty reasoning |
| 2 | Given an equation $x^{0,3} + x^2 - 1 = 0$ Find the solution to the equation using the bisection method by determining the initial bounds, constructing the algorithm, and computing each bound until obtaining the solution | <ul style="list-style-type: none"> - Find the bounds and prove it that contain a solution - Build the algorithm - Calculate each iteration | <ul style="list-style-type: none"> - Plausible reasoning - Memorize reasoning - Algorithmic reasoning |
| 3 | A pool with a volume of 14.04 m ³ , its height, length, and width are unknown. Find the surface area of the pool to be covered with tiles. | <ul style="list-style-type: none"> - Construct the model - Count the solution of "t" using bisection method - Using the proper bound - Find the surface area | <ul style="list-style-type: none"> - Mathematics foundation - Algorithmic reasoning - Plausible reasoning - Mathematics foundation |



The diagram shows a rectangular pool with blue tiles. Four labels are overlaid on the image: 'Volume=14.04 m³' at the top, 'Height = t cm' in the middle, 'Length = t+80' on the right side, and 'Wide = t+40 cm' at the bottom.

Construct a model so that we can find the value of "t" using the bisection method

Validity and reliability tests were conducted prior to the use of the test instrument, based on (Cohen et al., 2007). Content validation was selected and found appropriate for assessing the test instrument's validity in the context of numerical methods for solving non-linear equations, particularly the bisection method. The reliability test evaluated how the provided information context influenced students' performance in answering questions. The effect is proved by test, it

is formally formulated through the null hypothesis (H_0) and the research hypothesis (H_1) as follows:

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

Where μ_1 represents the mean score of pre-tests and μ_2 represents the mean score of post-tests on students' mathematical reasoning abilities from the population of students who received computer-assisted discovery learning via the web. Other instruments are (1) validation sheets, (2) observation sheets, and (3) interview scenarios. The data used in R&D can vary greatly depending on scope of the research, including qualitative, quantitative, secondary data, experimental, and observational data (Smith & Ragan, 2005). The data were analyzed using the data analysis approach based on Borg and Gall (1989), a systematic approach for data analysis in educational research, which includes data collection, data organization, data reduction, data presentation, conclusion drawing and verification, and data interpretation. The data obtained is in the form of qualitative data, namely suggestions, opinions and comments from expert observers and students on the feasibility of applying the web as computer assisted discovery learning, while the quantitative data is the results of the pretest-posttest of students' mathematical reasoning abilities, were analyzed by testing paired sample t-tests, assuming that the data is normally distributed and the variances of the populations are homogeneous.

Addie is an acronym for its five main phases: Analysis, Design, Development, Implementation, and Evaluation. Analysis is the initial step, which involves identifying user needs and the environment as well as setting objectives. This step is obtained from academic student data, curriculum, teaching-learning activities, and implemented curriculum. The analysis results will serve as the basis for formulating product designs that will be developed to be competent if applied on a certain scale. Initial designs are developed based on expert reviews using validation questionnaires and feedback. Assessment using a 4-point Likert scale validation questionnaire is as follows (Arikunto, 2016)

| | | | |
|-------------------|-----|---------------|------------------------------|
| Strongly Agree | : 4 | (Very Valid | : Average score > 3.50) |
| Agree | : 3 | (Valid | : Average score 2.50 – 3.50) |
| Disagree | : 2 | (Invalid | : Average score 1.50 – 2.50) |
| Strongly Disagree | : 1 | (Very Invalid | : Average score < 1.50) |

The validation process is carried out by two experts, called validators, whose advice becomes the basis for media development and will enter the implementation stage if deemed valid (average score >2.50).

Findings

In the research procedure, two stages of research were implemented, namely developing computer-assisted learning media based on discovery learning using the web, then the web was used to see its effect in training mathematical reasoning abilities. Based on these two stages, the research results also produced two different data, namely the web development stage using the RnD model with the ADDIE method and the application of computer-assisted discovery learning to improve mathematical reasoning abilities.

Development of computer-assisted learning based on discovery learning

Analyze

The analysis results were obtained from student learning outcomes and tests of students' reasoning abilities. Student learning outcomes shown by the Achievement Index (IP) are an average of 3.57, namely in the honors category. The distribution of student learning outcomes was obtained by 20 people in the range 3.51-4.00 in the "with praise" category, 13 people in the range 2.76-3.50 in the "very satisfactory" category, and none below 2.75. Looking at the minimum score, 33 students obtained learning results above 3.04. Based on an analysis of students' learning needs in numerical methods courses using a questionnaire given to 33 students, it was found that 27 students or the equivalent of 82% felt enthusiastic about the learning process in lectures, especially in numerical methods courses. Furthermore, the direct question and answer revealed that most were interested in the numerical methods course because this course teaches finding mathematical solutions in a way that is different from analytical methods, and can use computers or programming languages to obtain results. Apart from that, programming languages that are applied in numerical methods can train them in building programs or building algorithms.

The enthusiasm that students have is not in line with the difficulties they face when understanding numerical methods courses. A total of 24 students or 72% found it difficult to participate in learning. Respondents gave various reasons for this difficulty, most of whom considered the material taught in numerical methods courses to be too complex, starting from understanding the concept of a method or algorithm, building code or programs, evaluating programs to be able to provide appropriate solutions. Another cause is that students do not have software or applications to build programs because the applications required are paid or are not compatible with the PC they own. A small percentage have difficulty because they feel that the time needed is not enough, they don't have time to ask questions, practice with guidance, only practice independently through assignments.

The learning media used to present lessons are usually in the form of Power point slide shows and whiteboards. The advantage of Power Point is that it can display image illustrations and be supported by instructional explanations using a whiteboard. However, this media does not support learning with student center learning. Apart from that, students have also used a learning management system (LMS) to support learning, namely collecting assignments using Google Classroom. Each lesson also uses modules provided by the lecturer, hopefully as the main learning resource. From the questionnaire questions, web media was never used in learning. Only 3 students thought they had ever used the web as a teaching medium, while 30 students or 91% thought they had never used the web in numerical methods courses. They only use the web as a supporting learning resource.

Before filling out the questionnaire, respondents asked whether web media could overcome the difficulties faced in learning numerical methods. The hope is that the website can help students understand the concept of a method or algorithm with various graphic illustrations and calculation steps. It is hoped that web media can be used to build code or programs with low laptop specifications or without installation and for free. With web media, you can optimize study time by providing time for students to discuss and carry out guided exercises. If these difficulties can be overcome, then students agree to use the web as a learning medium in

numerical methods courses. In accordance with the respondents' answers, 24 students or 72% hope to be able to use web-based media to overcome difficulties in learning numerical methods.

Design

Based on the analysis of student needs in learning numerical methods, web-based media is needed which is expected to be able to overcome learning difficulties. Web design as a computer-assisted learning media based on discovery learning. Web learning internalizes the steps of the discovery learning learning model, namely (1) Stimulation, (2) Problem identification, (3) Data collection, (4) Data processing, (5) Verification, (6) Generalization (Borg & Gall, 1989). Apart from that, to answer students' learning needs, the website will be equipped with illustrative images and a cloud -based programming language management application. The web design map is shown in Figure 3 for the PC monitor or horizontal display and the HP or vertical display.

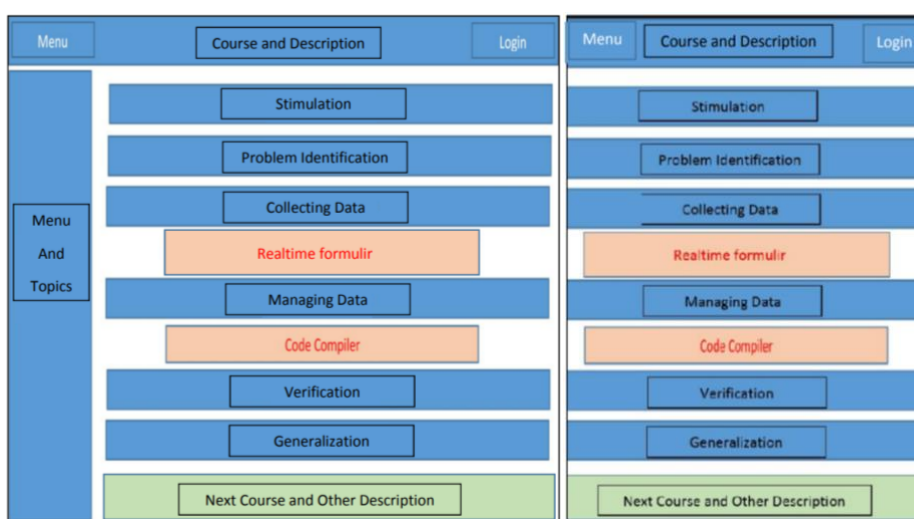


Figure 3. Web design map horizontal (left) and vertical (right) views

Each step of discovery learning is equipped with instructions as a guide in finding and illustrative images that can help students' understanding. The learning website is equipped with a Google form to collect student answers in real time as well as a cloud- based program development application using Octave online.

Develop

Based on the initial web design, a learning web will be built using HTML and CSS with internalization of discovery learning steps and equipped with Google Forms and Octave online applications. Obtained appearance discovery learning on the web is the main basis, the implementation of each discovery learning syntax can be accessed via the learning web.

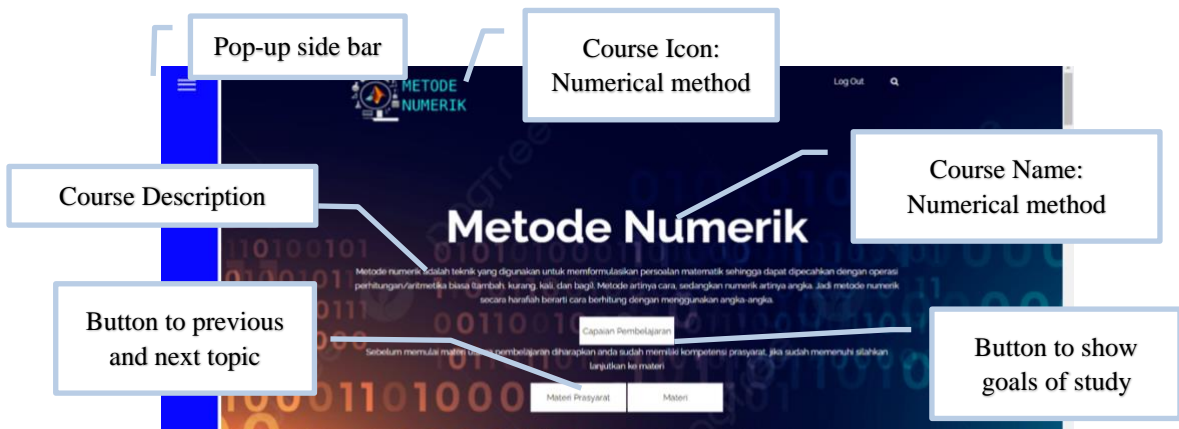


Figure 4. The home page appearance

Figure 4 displays a learning web page that explains the learning objectives, achievements, and steps or learning methods that will be implemented. The page tries to appear attractive, simple and easy to access. On the homepage, revisions provided by the validator include displaying learning objectives and CPL, adding a help menu in the sidebar, providing a summary of previous closely related material to upcoming discussions, and adjusting text color for better visibility on a dark background. These changes are evident in the redesigned learning website homepage, as shown in Figure 5.

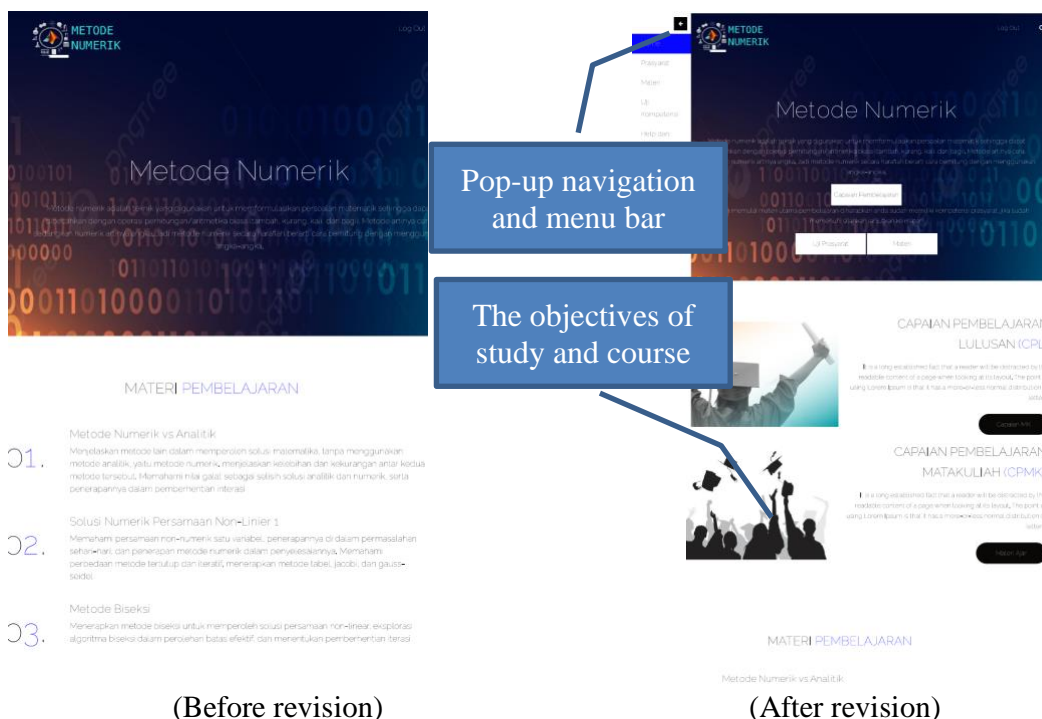


Figure 5. Revised features and content in the main page

Each discovery learning step is internalized on the web accompanied by a platform as hyper content that supports successful learning. Steps for discovery learning and media internalization on the web are presented below:

Providing stimulation

Stimulus or providing stimulation is by posing several problems that are closely related to everyday life, as in Figure 5, before and after revised: Before revised: The simulation interface merely provides examples of everyday problems, specifically in the field of economics, in determining the optimal value from the demand and supply equations.

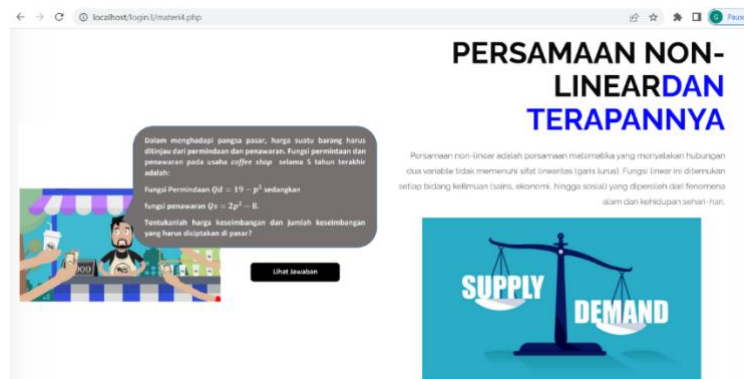


Figure 6. Stimulus from economic problem (before revision)

After revised: The display features everyday issues involving non-linear equations, as demonstrated in scientific articles.

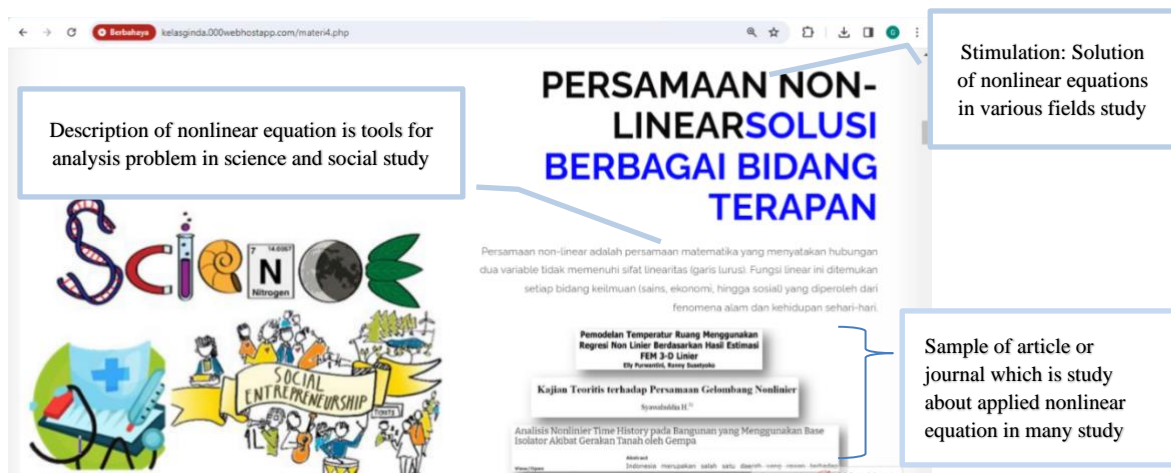


Figure 7. Stimulus with science articles related to nonlinear equations

In Figure 5, each reading source has an access link, making it easier for students to obtain articles related to non-linear equations as literacy material so that they are more interested in obtaining solutions to linear equations, thereby prompting students to recognize the significance of being able to derive solutions from non-linear equations.

Identification of problems

From one of the applications of nonlinear equations, problems are chosen that will bridge students' learning experiences. Before revised: identify problem through the problems provided in the stimulation section.

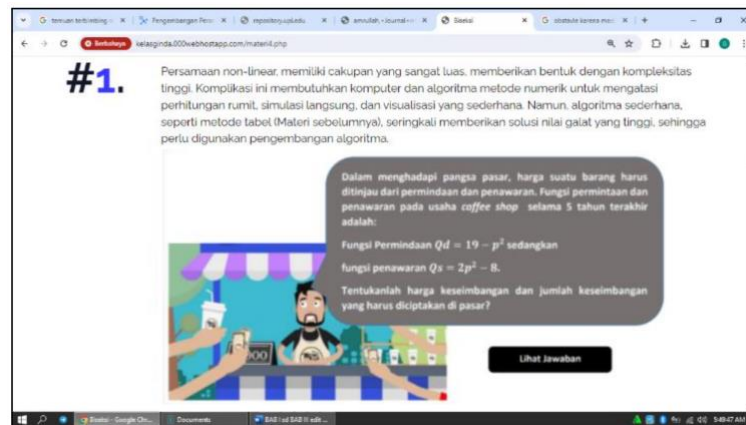


Figure 8. Problem identified by economic problem (Before the revision)

After the revision: The appearance of the problem to be identified is the trajectory of solid material in volcanic eruptions and simulated it to applet.

Figure 9. Problem identification by volcano phenomena (After revision)

Data collection

The data collection process is constructed in several steps, beginning with students determining and dividing the domain range containing solutions into two parts, up to building algorithms from the calculated results provided. On the web platform, determining the range containing solutions involves students being asked to approximate solutions to non-linear equations, as well as identifying the barrier containing the solution. Students can fill out a Google form so that responses can be monitored in real-time.

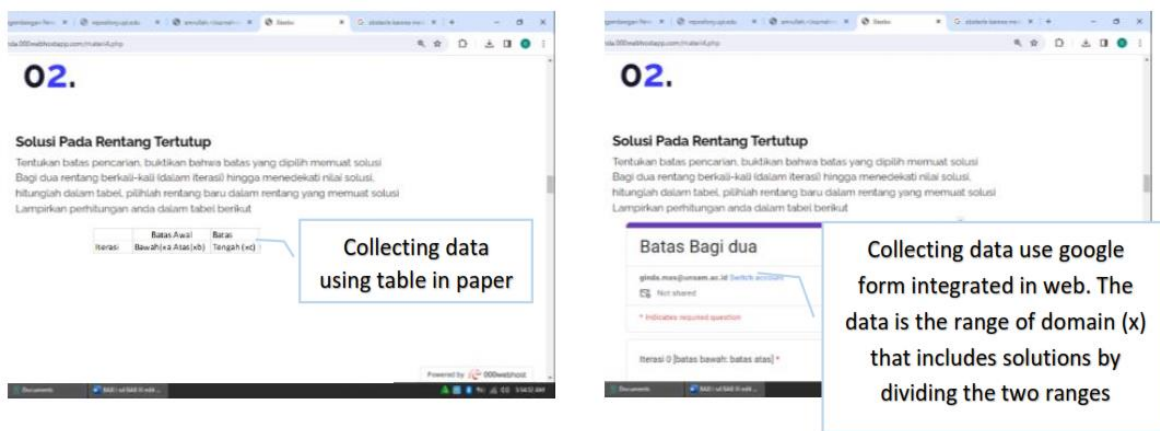


Figure 10. Collecting data using paper (Left panel); Data collection using google forms on the web (Right panel)

Students are asked to demonstrate the appropriate method used in determining the divider boundaries. Prior to the revision, this process involved brief questions that would be asked directly or written through Student Activity Sheets (SAS). This was addressed by utilizing a digital platform, namely integrating Google Forms.

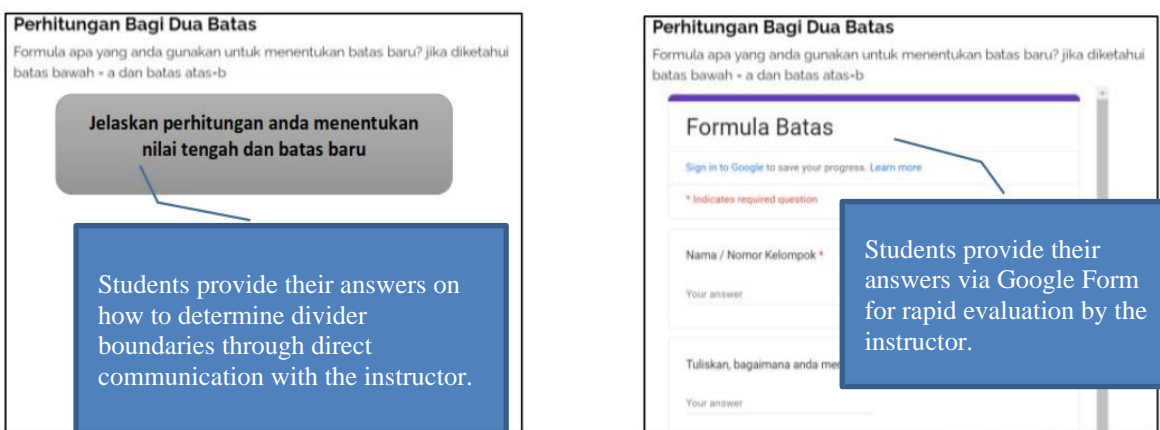


Figure 11. Data collection using paper (Left panel); using google forms on the web (Right panel)

It is expected that, from the data collection in the previous sub-steps, students will be asked to construct the appropriate algorithm. This process can be carried out in real-time, allowing instructors to promptly evaluate students' opinions or answers. Furthermore, students will be asked to observe the results of the bisection calculation and provide feedback in the form of systematic steps.

The calculation of every iteration from bisection method to prove range of domain (x) that includes solutions by dividing the two range can find nonlinear solution. Step to proof step "collecting data"

Students observe each iteration of the calculation from the provided table and are asked to construct steps accordingly

This page has been revised to provide a column for students' answers or opinions, directly integrated and monitored by the instructor.

| Iterasi | Batas Awal | Batas | Nilai Fungsi | | | Batas Baru | | Error | | |
|---------|--------------------|-------------|--------------|----------|----------|--------------|--------------|----------|----------|----------|
| | Bawah(xa Atas(xb)) | Tengah (xc) | f(xa) | f(xb) | f(xc) | xa | xb | | | |
| 0 | 10 | 80 | 45 | 14,12051 | -66,2359 | 13,14228634 | 185,5757604 | 45 | 80 | 13,14229 |
| 1 | 45 | 80 | 62,5 | 13,14229 | -66,2359 | -16,74682453 | -220,0915632 | 45 | 62,5 | 16,74682 |
| 2 | 45 | 62,5 | 53,75 | 13,14229 | -16,7468 | 0,647730907 | 8,512665049 | 53,75 | 62,5 | 0,647731 |
| 3 | 53,75 | 62,5 | 58,125 | 0,647731 | -16,7468 | -7,43704681 | -4,817205074 | 53,75 | 58,125 | 7,437047 |
| 4 | 53,75 | 58,125 | 55,9375 | 0,647731 | -7,43705 | -3,241532952 | -2,099641078 | 53,75 | 55,9375 | 3,241533 |
| 5 | 53,75 | 55,9375 | 54,84375 | 0,647731 | -3,24153 | -1,258619772 | -0,815246977 | 53,75 | 54,84375 | 1,25862 |
| 6 | 53,75 | 54,84375 | 54,296875 | 0,647731 | -1,25862 | -0,29587412 | -0,191646812 | 53,75 | 54,29688 | 0,295874 |
| 7 | 53,75 | 54,29688 | 54,0234375 | 0,647731 | -0,29587 | 0,178320971 | 0,115504005 | 54,02344 | 54,29688 | 0,178321 |
| 8 | 54,02344 | 54,29688 | 54,1601563 | 0,178321 | -0,29587 | -0,05817843 | -0,010374434 | 54,02344 | 54,16016 | 0,058178 |
| 9 | 54,02344 | 54,16016 | 54,0917969 | 0,178321 | -0,05818 | 0,060220807 | 0,010738633 | 54,0918 | 54,16016 | 0,060221 |
| 10 | 54,0918 | 54,16016 | 54,1259766 | 0,060221 | -0,05818 | 0,001058573 | 6,37481E-05 | 54,12598 | 54,16016 | 0,001059 |
| 11 | 54,12598 | 54,16016 | 54,1430664 | 0,001059 | -0,05818 | -0,028550583 | -3,02229E-05 | 54,12598 | 54,14307 | 0,028551 |
| 12 | 54,12598 | 54,14307 | 54,1345215 | 0,001059 | -0,02855 | -0,013743669 | -1,45487E-05 | 54,12598 | 54,13452 | 0,013744 |
| 13 | 54,12598 | 54,13452 | 54,130249 | 0,001059 | -0,01374 | -0,006341964 | -6,71343E-06 | 54,12598 | 54,13025 | 0,006342 |

Figure 12. Collecting data and processing to build the algorithm (before to after the revision)

Verification

The verification step is carried out through two activities: the compilation of algorithms from the steps and opinions provided by each group of students, and their substantiation. Verification can be performed by posing agreement or disagreement questions to the students, and their direct answers are verified with appropriate reasoning. Each chosen step is substantiated through simulation using a Geogebra applet.

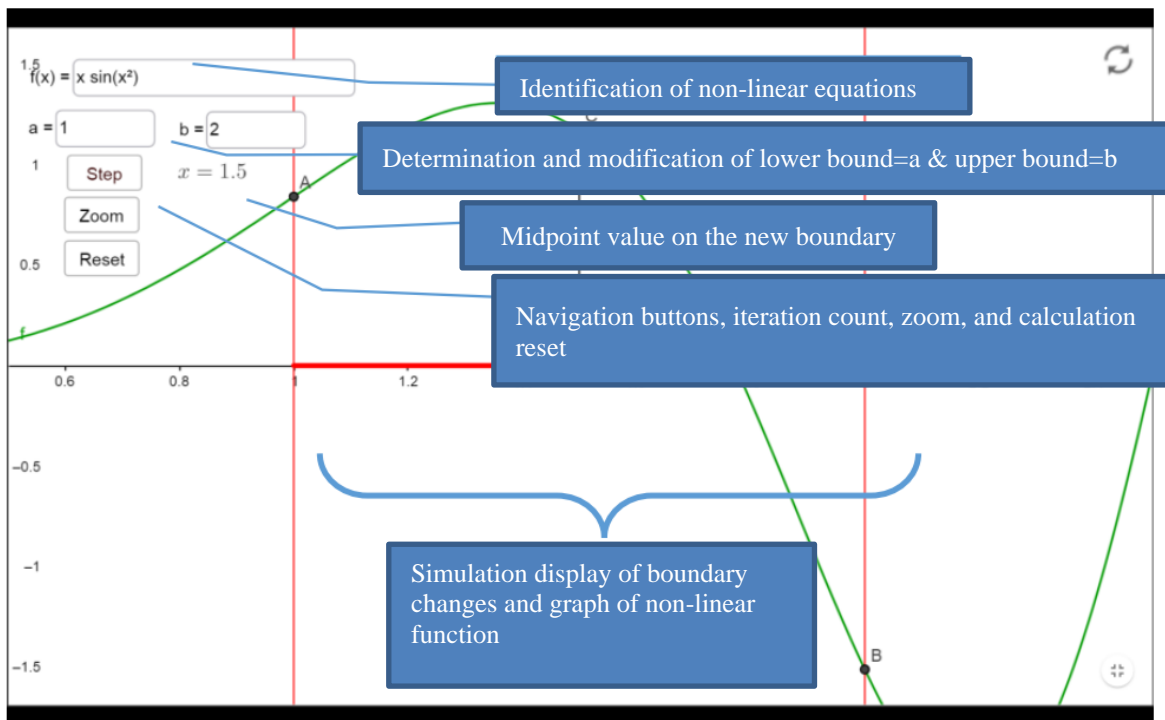


Figure 13. Applet of GeoGebra to make verification

The lecturer guides the students to draw conclusions, thereby building a bisection algorithm based on learning experience. The five series of learning steps are internalized on the learning web by combining various platforms, so that computer-assisted discovery learning can be implemented and help the learning process.

Implement

Application of the web as computer-assisted learning based on discovery learning to 12 students outside the research sample. Learning is carried out by forming groups of students consisting of 3 people and are asked to access the web and follow the lecturer's guidance.



Figure 14. Students access the learning website using cellphones (Left panel) and Laptops (Right panel)

Computer-assisted learning based on discovery learning using the web is applied in numerical methods courses, in the material of determining solutions to linear equations using

the bisection method. Apart from that, the web is also applied to online and offline classes separately and simultaneously. Based on the evaluation results, the application of the learning web has improved and is proven to help student learning activities.

Evaluate

The web as a medium for computer-assisted learning based on discovery learning is evaluated using various data collection techniques in the form of questionnaires, interviews and direct observation. From this data, the aspects that will be reviewed are to see the clarity of the computer-assisted learning web based on discovery learning, the suitability of the material presented, the suitability of the discovery learning model, and the accuracy of the web as a computer-assisted learning medium based on discovery learning as a whole. Computer-assisted learning using the web is applied to mathematics education students in numerical methods courses, especially in material on numerical solutions of non-linear equations using the bisection method. The results of the evaluation and obtaining improvements based on these aspects are as follows;

a. Clarity of computer-assisted learning web based on discovery learning

Web clarity is obtained based on assessments from two web media and learning media experts. Informatics lecturer as a web media expert (Validator-1/V1) and mathematics education lecturer with doctoral education as a mathematics learning media expert (Validator-2/V2). Based on the results of the evaluation provided and obtaining input for improvements, they are as follows:

Table 2. Evaluation the clarity of web

| No | Assessment Aspects | Repair | Score | | | |
|----|--|---|-----------|----|----------|----|
| | | | Beginning | | Repaired | |
| | | | V1 | V2 | V1 | V2 |
| A | Graphic Display | | | | | |
| 1 | The front-page display is related to learning | Display learning objectives and CPL | 3 | 3 | 4 | 4 |
| 2 | Graphic display of each web page | Use images that are appropriate and do not become a distraction | 3 | 2 | 4 | 4 |
| 3 | Complete material according to the table of contents | Provide a help menu in the sidebar, provide a summary of previous material that is closely related to the material that will be discussed next. | 4 | 2 | 4 | 4 |
| 4 | The font size is appropriate, and the color is clearly legible | Adjust the color on a dark background, there is still writing that is difficult to see | 2 | 3 | 4 | 4 |
| B | Multimedia | | | | | |
| 5 | The media used is appropriate to explain the material | In the “data collection” section, it is not enough to just use images. Use other media such as videos or applets | 2 | 3 | 4 | 4 |
| 6 | The illustrations presented are interesting | Look for other illustrations that fit everyday life | 3 | 2 | 4 | 4 |
| 7 | The media used is varied | Make more use of the Geogebra Applet, not just pictures or videos | 2 | 2 | 4 | 3 |
| | Practicality of operation | | | | | |

| | | | | | | |
|---|--|---|-----|-----|---|-----|
| 8 | There are instructions for using the website | Add instructions for use | 1 | 1 | 4 | 3 |
| 9 | Ease of access | Can be accessed by PC and cellphone with a customizable display | 2 | 1 | 4 | 4 |
| | | Average | 2.5 | 2.4 | 4 | 3.8 |

Both of validator give responses which is score and comments is showed in [Table 2](#). Score before and after revised are 2.5 to 4 from first validator and 2.4 to 3.8 from second validator, it means both of validator are not satisfaction and website need to revise according to their comments. First Validator (V1): Answers or responses from students are not collected in real-time using the system. Lecturers still do direct questions and answers, so it takes a lot of time. Each student's answers can be input via the HTML database (localhost) or Google form. Second Validator (V2): Use examples that are relevant to everyday life or can be simulated. Because the website can attach media from other websites, complete this website with videos from YouTube or Geogebra applets. Website will get some additional feature, such us lecturer's learning videos, applets or use ones that are already available.

b. Suitability of the material presented

The material suitability aspect was tested through two validators, namely material experts by applied mathematics lecturers who have taught numerical methods for more than three years, while mathematics learning experts by experienced mathematics education lecturers who hold doctorates. Each of these experts is first validator (V1) and second validator (V2).

Table 3. Evaluation of content suitability

| No | Assessment aspects | Repair | Score | | | |
|----|--|--|-----------|----|----------|----|
| | | | Beginning | | Repaired | |
| | | | V1 | V2 | V1 | V2 |
| A | Appropriateness of material/content | | | | | |
| 1 | Material conformity with CPL | Adjust to the study program CPL at the research location. | 4 | 3 | 4 | 4 |
| 2 | Suitability of material to learning objectives | The objective specification is only on bisection material | 2 | 3 | 4 | 4 |
| 3 | Material accuracy | Complete with prerequisite material, namely limits for containing solutions and approximate values before discussing bisection | 1 | 3 | 3 | 4 |
| 4 | The material uses appropriate examples | Examples are accompanied by illustrations, not just narration | 3 | 2 | 4 | 4 |
| 5 | Short, concise and clear material | Use clear and precise language | 3 | 3 | 4 | 4 |
| B | Feasibility of presentation | | | | | |
| 6 | Presentation of coherent/coherent concepts | Include prerequisite materials | 2 | 3 | 3 | 4 |
| 7 | Availability of exercises | Provide guided practice and independent practice. Practice using everyday problems | 2 | 2 | 4 | 3 |

Special note from validator I: Give an explanation of the solution loading limit in the closed method, do not go directly to the bisection method or explain it after the algorithm. It should be

reminded again that numerical methods produce solutions in the form of approximate values, not true values, so they always have errors. If possible, explain the convergence of the iteration method. Validator II: There is still instructional language that is inaccurate, due to typing errors or is not standard. Apart from that, instructions that are too long will make it difficult for readers.

Suitability to the discovery learning model

Web learning internalizes the steps of the discovery learning model, namely (1) Stimulation, (2) Problem identification, (3) Data collection, (4) Data processing, (5) Verification, and (6) Generalization. The aspect that evaluates the feasibility of the discovery learning model is two mathematics education lecturers with doctoral education and who have more than five years of teaching experience. The evaluation results are as follows:

Table 4. Suitability of discovery learning model

| No | Assessment aspects | Repair | Score | | | |
|----|--|--|-----------|----|----------|----|
| | | | Beginning | | Repaired | |
| | | | V1 | V2 | V1 | V2 |
| 1 | Accommodate stimulation levels appropriately | Stimulation uses examples from everyday life or from research journals to open students' insight into the benefits of learning to foster a sense of interest | 3 | 3 | 4 | 4 |
| 2 | Instructions The problem identification stage is appropriate | The problems identified are based on real cases with simpler calculations, not too difficult | 2 | 3 | 4 | 4 |
| 3 | The data collection stage is efficient and effective | Data collection can use a digital platform , so it is real time without waiting for students to answer one by one | 3 | 3 | 4 | 4 |
| 4 | Verification stage instructions are precise | Verification can use Google Form so that students can provide opinions directly without being asked first | 3 | 3 | 4 | 4 |
| 5 | Generalization instructions are appropriate | Direct students to build conclusions and generalizations | 2 | 2 | 3 | 4 |

Special note get from Validator I that the stimulation and identification stage, problems are used that students can imagine or occur in everyday life. Complete with appropriate illustrative images or videos. The use of Google forms at the data collection and verification stages can be considered to shorten learning time. Validator II: Students will find it difficult to explain the results of data collection and verification if they are only asked directly, generally in the field students are reluctant to express their opinions. Make sure students gain learning experience so they are able to build appropriate generalizations.

The accuracy of the web as a computer-assisted learning media based on discovery learning

Web was evaluating from 12 students' experience when using it to learn, in the entire questionnaire obtained the following results:

Table 5. User experience when using web to study

| No | Aspects/Questions | Agree | No | Note |
|----|--|-------|----|---|
| 1 | The learning web is easy to use in learning | 7 | 5 | - |
| 2 | Web learning can shorten time | 8 | 4 | Students feel that learning will be shorter if the lecturer explains the solution directly in front of the class and the students pay attention |
| 3 | Easy to access anytime and anywhere | 8 | 4 | Requires quota and internet network, some students cannot access the internet at home |
| 4 | The description of the material and exercises is easy to understand | 10 | 2 | It's easy to understand if you give illustrations and story questions that aren't too long |
| 5 | Attractive web appearance | 4 | 8 | It is less attractive to some students because the colors are monotonous or do not suit personal preferences |
| 6 | The content on the website is related to images, tables and diagrams in accordance with the material | 11 | 1 | - |
| 7 | The language used is easy to understand | 12 | 0 | - |
| 8 | The color combinations used are appropriate and attractive | 8 | 4 | The appropriate color combination does not disturb the writing, but is less attractive |
| 9 | The web helps independent learning | 10 | 2 | Very helpful, even though it feels inefficient and still requires guidance from the lecturer |
| 10 | Feel helped after using web media | 9 | 3 | It helps because there are independent exercises and the answers can be verified directly |
| 11 | The web is useful for increasing insight | 11 | 1 | - |

Implementation of the computer-assisted learning based on discovery learning to improve mathematical reasoning ability

Data was obtained by pre-posttest design research involving 33 students, namely providing treatment in the form of computer -assisted discovery learning using the web. This stage provides value for the acquisition of mathematical reasoning abilities and improvement of students' mathematical reasoning abilities.

Table 6. Descriptive analysis of gains and improvements

| Statistical aspects | | | Reasoning approach | | N-gain reasoning | |
|----------------------------------|-------------|--|--------------------|------------|------------------|------------|
| | | | Statistics | Std. Error | Statistics | Std. Error |
| Mean | | | 63.8003 | 3.13770 | .5809 | .03116 |
| 95% Confidence Interval for Mean | Lower Bound | | 57.4090 | .5174 | | |
| | Upper Bound | | 70.1916 | .6444 | | |
| 5% Trimmed Mean | | | 65.1718 | | .5925 | |

| Statistical aspects | Reasoning approach | | N-gain reasoning | |
|---------------------|--------------------|------------|------------------|------------|
| | Statistics | Std. Error | Statistics | Std. Error |
| Median | 65.4500 | | .5600 | |
| Variance | 324,889 | | .032 | |
| Std. Deviation | 18.02469 | | .17900 | |
| Minimum | 16.36 | | .12 | |
| Maximum | 87.27 | | .83 | |
| Range | 70.91 | | .71 | |
| Interquartile Range | 19.09 | | .22 | |
| Skewness | -1.157 | .409 | -.921 | .409 |
| Kurtosis | 1.231 | .798 | .828 | .798 |

The results of data analysis obtained post-test scores for mathematical reasoning abilities obtained various statistical aspects. The average value obtained for learning with computer assisted discovery learning is 63.8003. Based on the standard deviation (Std. Deviation = 18.02) and the level of inclination or slope (Skewness = -1.157). It tends to be negative, meaning that the acquisition of mathematical reasoning abilities tends to converge on a large score. The results of data analysis increased mathematical reasoning abilities, namely with an average value of learning with computer assisted discovery learning of 0.5809. Based on the average value, it is also shown that the increase in the criteria is moderate. Based on standard deviation (Std.deviation=0.179) and the level of inclination or slope of increased (Skewness = -0.921), meaning that the increase in reasoning ability spreads with a tendency to converge on a large score.

Table 7. Paired sample t-test for mathematical reasoning ability

| | | Paired differences | | | | t | df | Sig. (2-tailed) | |
|--------|-------------------------------|--------------------|----------------|-----------------|---|-----------|---------|-----------------|-------|
| | | Mean | Std. deviation | Std. error mean | 95% Confidence interval of the difference | | | | |
| | | | | | Lower | | | | Upper |
| Pair 1 | Pre_Test_DL - Post_Test_DL | 48.42975 | 14.43007 | 2.51195 | -53.54643 | -43.31307 | -19.280 | 32 | .000 |

Obtaining mathematical reasoning ability scores is paired (paired) pretest and posttest for each lesson. The first pair (pair- 1), namely the pretest and posttest in learning with computer-assisted discovery learning, obtained a significant level of 0.00, this value is a number smaller than $\alpha = 0.05$. This shows that the hypothesis related to the application of computer assisted based discover learning has a significant influence on the acquisition of mathematical reasoning abilities and can be accepted.

Table 8. Paired samples correlation test

| | | N | Correlation | Sig. |
|--------|----------------------------|----|-------------|------|
| Pair 1 | Pre_Test_DL & Post_Test_DL | 33 | .609 | .000 |

The correlation value for the pretest and posted pairs of computer-assisted discovery learning is 0.609 with a significance level of 0.05. This can be interpreted as that computer assisted based discover learning has a significant positive influence between the pretest and posttest of students' mathematical reasoning abilities.

Discussion

The research results obtained two parts of data, namely at the web development stage as a computer-assisted learning media based on discovery learning and its influence on improving students' mathematical reasoning abilities.

Web development as a computer-assisted learning media based on discovery learning

Mathematics learning media using web-based discovery learning in numerical methods courses, material on solutions to nonlinear equations using the bisection method, is validated based on several evaluation aspects. Based on the clarity of the computer-assisted learning web based on discovery learning, the average score obtained from the two validators who were considered experts was valid to be used in teaching. The score of evaluation and comment shows that the web as computer-assisted learning based on discovery learning is very suitable for presenting material in numerical methods courses, especially numerical methods. The results of content validation are in line with several previous studies. Research by Martín-Caraballo et al. (2015) shows that geometric visualization is needed in building non-linear equation solution algorithms, applets on the web can help interpret geometry using the bisection method. Handayani et al. (2017) also carried out a similar evaluation and obtained the results of the development of teaching material software assisted by content valid numerical methods, the assessment of the numeric method teaching material validator was good so that it could be used with slight revisions.

In line with the suitability of the material, the learning web is in accordance with the discovery learning model, which is viewed from each learning stage. The web was declared to be in accordance with discovery learning, namely based on an assessment that obtained 2.7 in the valid category and increased in the very valid category with a score of 3.9. Validation of the computer-assisted learning web based on discovery learning was carried out in stages, with the next stage being validation of the accuracy of the web as a whole through questionnaires and interviews with students and several teaching lecturers. Based on the results of questionnaires to students, it was found that the majority of students agreed that the website helps learning with a display that suits the material, easy language, supports independent learning and is easy to use. However, there are still many students who think the web display is less attractive because the monotonous colors do not suit their personal wishes. Apart from that, students think that the web will not be able to shorten learning time. This is not in line with interviews with several lecturers, web learning helps the learning process become meaningful compared to only using instructional learning, without using computer-assisted web learning based on discovery learning.

Some of the revisions implemented include explaining the learning objectives and learning outcomes with the aim of increasing student learning motivation and students knowing the learning targets to be achieved. Use images or illustrations that are more relevant and do not cause diversion of learning objects. Apart from that, the media used is more varied, namely utilizing Geogebra applets that are readily available and relevant to learning instructions. Web media can also be accessed via smart phone because the majority of mathematics education students use smart phones, and find it bothersome if they always have to use a laptop in learning. From the results of the evaluation of the computer-assisted learning web based on discovery

learning, improvements and increased validity of the web in computer-assisted learning based on discovery learning were obtained.

The influence of computer-assisted discovery learning using the web on students' mathematical reasoning abilities

Statistical analysis shows that computer-assisted discovery learning using the web has a positive influence on improving students' mathematical reasoning abilities. Based on pretest and posttest data on mathematical reasoning abilities, the effect of computer-assisted discovery learning using the web applied on students' mathematical reasoning ability scores. The relationship between learning and the acquisition of mathematical reasoning abilities will be shown by the magnitude of the influence (correlation) is 0.609 with a significance level of 0.05. This can be interpreted as that computer assisted based discover learning has a significant positive influence between the pretest and posttest of students' mathematical reasoning abilities.

The results of the analysis, the output of the paired sample t-test and the paired sample correlation test. In line with several previous studies, namely the application of digital media improves mathematical reasoning abilities in seventh grade students' geometry material (Nurjannah et al., 2020). Discovery learning-based learning media can improve mathematical problem-solving abilities (Suratno et al., 2018). The problem-solving process involves mathematical reasoning abilities, so that increasing problem-solving abilities is in line with the increase in mathematical reasoning described in Hidayatullah (2019). Robová (2013) further stated that the use of web learning in the mathematics teaching and learning process provides teachers with information on how to motivate and attract students, increase the clarity of explanations and include proactive methods in the classroom.

A series of study findings is confirmed by the significant influence of implementing web-based learning ($\text{Sign} = 0 < \alpha = 0.05$) on mathematical reasoning outcomes and improvement, with a correlation ($r = 0.609$) falling into the moderate category. The relationship between support from digital media based on discovery learning and the positive effect on mathematical reasoning skills, as obtained from study results (Bond et al., 2020; Zhang et al., 2019), provides an overview of how multimedia technology potentially supports the development of students' reasoning abilities.

Conclusion

The conclusion drawn from the study is that web-based learning validated for use in computer-assisted discovery learning. Based on the constructed web media, it can be organized to involve various multimedia, not only images or videos, but also simulation features, namely using Geogebra applets. The recommendation from validators is the integration of tools into the web, which has been tested and validated for implementation. This process is supported by web development techniques and its integration into discovery learning, providing users with a more interactive and in-depth learning experience. Evaluations of these media suggest their implementation within web-based learning. The appropriate use of media enhances success in learning, thus supporting the improvement of reasoning skills. Studies supporting this have been conducted, demonstrating that web-based digital media built with appropriate relevance to the material supports discovery learning, which can increase student engagement and consequently support the enhancement of their reasoning abilities.

The limitations of this research include the research subjects, namely the mathematics group at the university level, in the numerical methods course. The learning basis used is discovery learning integrated with features in digital media in the form of the web. The application is carried out as an effort to improve mathematical reasoning abilities. Recommendations for further research include enhancing the performance of web-based learning with other instructional models or methods. Expanding the research sample to different educational levels. Expanding the cognitive object of the study to include high order thinking skills.

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