Beta: Jurnal Tadris Matematika, 16(1) 2023: 38-54
DOI 10.20414/betajtm.v16i1.550

Research articles

Teachers' Pedagogical Content Knowledge (PCK) in implementing Realistic Mathematics Education (RME)

Tuti Zubaidah[1], Rahmah Johar[1], Dewi Annisa[1], Yuhasriati[1], Yulinar Safitri[2]

Abstrak *Pedagogical Content Knowledge (PCK)* atau pengetahuan pedadogis dan konten sangat diperlukan bagi guru matematika dalam pembelajaran karena matematika bersifat abstrak dan saling berkaitan, sehingga memerlukan penguasaan materi dan perencanaan pembelajaran yang baik. Pembelajaran dengan Pendidikan Matematika Realistik (RME) membantu siswa belajar bermakna karena dimulai dari masalah kontekstual. Oleh karena itu, untuk mewujudkan pembelajaran bermakna tersebut, guru harus memiliki PCK dalam menerapkan RME. Penelitian ini bertujuan menganalisis PCK dua orang guru dalam menerapkan RME pada materi geometri. Kedua guru tersebut telah mengikuti program pengembangan profesional guru sebelum pembelajaran dilaksanakan. Data dalam penelitian ini dikumpulkan melalui observasi dan wawancara yang dianalisis secara kualitatif. Hasil penelitian menunjukkan bahwa kedua guru memiliki PCK yang sama dalam menerapkan RME. Indikator yang dipenuhi dibahas lebih lanjut dalam tulisan ini.

Kata kunci *PCK, RME, Pengembangan profesional guru, Geometri*

Abstract Pedagogical Content Knowledge (PCK) is crucial for mathematics teachers in teaching and learning due to the fact that mathematics is an abstract and interrelated subject; requiring adequate knowledge on the content and instructional practices. Applying Realistic Mathematics Education (RME) helps students learn in meaningful way since it starts with contextual problems. Therefore, to realize the meaningful learning, teachers need to have PCK in implementing RME. This study aims to analyze two teachers' PCK in implementing RME for a geometry topic. Prior to the classroom teaching, both teachers received teacher professional development program regarding RME. Data in this study were collected through classroom observations and interviews which were the analyzed qualitatively. The results of the analysis show that both teachers have similar PCK in applying RME. The PCK indicator fulfilled by the teachers is further discussed in this paper.

Keywords *PCK, RME, Teacher professional development, Geometry*

Introduction

The quality of teachers is an essential factor in mathematics teaching and learning to achieve learning goals (Gess-Newsome et al., 2017). Professional teachers possess skills to integrate and transform knowledge and deliver it to make students easily comprehend (Ma'rufi et al., 2018). Professional teachers are described as the three pillars of knowledge: content knowledge (CK), pedagogical knowledge (PK), and pedagogical content knowledge (PCK), an intersection

[1] Mathematics Education Department, Universitas Syiah Kuala, Indonesia, tutizubaidah@usk.ac.id
[2] Realistic Mathematics Education Research Centre, Universitas Syiah Kuala, Indonesia

between Content Knowledge (CK) and Pedagogical Knowledge (PK) (Shulman, 1987; Baumert & Kunter, 2006).

Shulman (1987) first introduced the concept of Pedagogical Content Knowledge (PCK). PCK is a fundamental knowledge base component for carrying out learning activities. The teacher is not just to deliver material but also to develop educational activities and help students with learning difficulties. Teachers with qualified PCK can align knowledge to make learning more comprehensive (Ma'rufi et al., 2018). Otherwise, the teacher could have difficulties teaching mathematics effectively.

Teachers with adequate PCK can guide students to achieve higher scores in mathematics learning (Cueto et al., 2017; Callingham et al., 2016). Mathematics teachers with high PCK can effectively transfer their knowledge and skills to students (Hill et al., 2008). Hence, Mathematics teachers must master mathematical concepts and how to convey the concept to students and lesson plans, as mathematics is abstract and interrelated (Kula-Ünver, 2020; Manizade & Mason, 2011; Sarwah & Ilyas, 2019; Tanışlı et al., 2020). With the existence of PCK, students can easily understand the learning material. Considering its function, PCK shall be more beneficial and developed as mathematics learning requires good learning systematization and optimal teacher performance in planning and implementing learning activities (Nugraha et al., 2020).

Admittedly, teacher PCK in learning can lead to student misconceptions about teaching materials or the emergence of student reluctance (Fiangga et al., 2020). A further instance occurs in geometry (Guncaga et al., 2017). The most common errors are triangles, squares, and circles. Having little experience with manipulation becomes the primary cause. Thus, it is essential to provide models and non-models of these forms (Kadarisma et al., 2020). In addition, discussing the geometry function, particularly in determining the definition, cannot be limited to introducing the shape's name. In addition, this misconception is caused by a need to understand the concepts and principles of each plane geometry. Therefore, it is difficult to understand the relationship between each plane geometry and its definition. Students needed better comprehension in applying formulas and had to write units in answers, for example, length in centimeters (cm) (Sopiany & Rahayu, 2019).

Based on the description of this misconception, designing learning trajectories needs a learning innovation that can support students to understand the concepts in geometry material. The learning trajectory is a learning design that considers students' thinking levels (Andrews-Larson et al., 2017; Rich et al., 2018), where students learn in their way, become active, and create their knowledge continuously. The learning trajectory describes students' thinking through various activities to achieve learning goals. Through this activity, students must understand concepts and see the meaning of the material and its relation to daily life (Buelow et al., 2018; Tanujaya et al., 2021). Therefore, one of the learning innovations in designing a learning trajectory is applying the Realistic Mathematics Education (RME) approach. Learning with RME allows students to rediscover and build mathematical concepts based on realistic problems presented by the teacher (Majid, 2017; Afriansyah, 2021). Realistic situations in learning allow students to use their informal knowledge to solve problems (Sumirattana et al., 2017). The RME applies real-world contexts in transferring the lesson (Edo & Samo, 2017). Students are expected to have better motivation because they think mathematics is closely related to the real world.

Theoretical Review

Realistic Mathematics Education (RME)

RME is one approach that begins with a real problem. At first, the students solve solely and continue to be monitored until it arrives at the stage of formal thinking through the process of horizontal mathematicalization and vertical mathematicalization (Gravemeijer, 1994; van den Heuvel-Panhuizen & Drijvers, 2014). Freudenthal (1983), the founder of RME, explains that in the RME concept, students must reinvent mathematical ideas, including concepts, strategies, procedures, formulas, and definitions, with the teacher's guidance because mathematics is a human activity, not a ready-made.

Gravemeijer (1994) mentions three principles of RME: guided reinvention and progressive mathematization, didactical phenomenology, and self-developed models (models built by students). At first glance, students are given the same opportunity to build and rediscover mathematical ideas and concepts, meaning that learning starts from contextual or actual problems. Subsequently, the "progressive process" emerges, which involves two steps of mathematics, namely horizontal and vertical mathematics, starting from a given contextual problem and finishing with formal mathematics. The second is didactic phenomena; the mathematical topics come from phenomena that occur in daily life or imaginable problems. The third principle of self-developed models is that students build their models ('model of' and 'model for'), which begin with the actual situation. Furthermore, Treffers (1978) proposed five characteristics of RME, starting learning with real problems, using a model, using student construction results, interactivity, and intertwinement.

Mathematics learning with the RME approach includes the following four aspects (de Lange, 1995):

- Learning by asking real problems based on students' experience and level of knowledge so that they are involved in learning meaningfully.
- The problems given must be directed following the goals of the learning process.
- Students create or develop symbolic models informally based on the problems.
- The learning process is conducted interactively. The student explains and gives reasons for the answers he gives, understands the answers of his friends (other students), agrees with his friend's answers, expresses disapproval, seeks other alternative solutions, and reflects on each step taken or on the results of the lesson.

Using the RME approach allows students to develop their knowledge under the teacher's guidance and rediscover mathematical ideas that experts have discovered. The activity of rediscovery of mathematical ideas begins with actual problems. Thus, the teacher guides students to arrive at a more formal or abstract stage of mathematical knowledge.

Learning using the RME approach starts with a real problem, and students find informal solutions to real problems. These problems can be in the form of story problems in everyday life or problems that students can imagine, such as models, drawings, sketches, or mathematical problems. Moreover, students will start to solve problems that demand higher, broader, or more complicated mathematical abilities such as formulas, understandings, and algorithms. The teacher examines and discusses each student's strategy during class (van den Heuvel-Panhuizen & Drijvers, 2014).

Pedagogical Content Knowledge (PCK)

Shulman (1987) describes PCK as a blend of all-encompassing content and pedagogy needed to teach a subject or topic in a way that makes it understandable to others. PCK is a characteristic of the teacher's knowledge of how to work on learning materials (Koh et al., 2010). According to van Driel et al. (2002), PCK includes subject matter, general pedagogical, and contextual knowledge. PCK is teaching knowledge where teachers can interpret knowledge to facilitate student learning (van Driel et al., 2002). Therefore, PCK is the teacher's interpretation and transformation of subject matter knowledge. The indicators of PCK teachers in applying the RME approach were adapted from Maher et al. (2015).

Table 1. The category and indicator of PCK

No.	PCK category	Indicator
1	*Clearly PCK*	
	a. Teaching and learning strategy	Discussing/using strategies or approaches in teaching mathematical concepts
	b. Student thinking	Discussing/directing students to think about concepts or trying to help students to be at a certain level of understanding
	c. Student thinking -a misconception.	Discussing/helping students who have misconceptions
	d. Cognitive demands of a task	Identifying the task aspects affecting the cognitive level (C1, C2, and so on)
	e. The suitability of the presentation with the characteristics of the concept	Explaining/demonstrating concepts using models or illustrations (e.g., props, diagrams, etc.)
	f. Knowledge of learning resources	Discussing/using learning resources during learning
	g. Knowledge of curriculum	Discussing how the topics taught are in accordance with the curriculum (based on previous and future grades)
	h. Learning objective	Discussing the relationship of the topic/subject with other relevant materials or applying the concept
2	*The ability to understand concepts concerning pedagogical abilities*	
	a. Excellent understanding of basics mathematics	Demonstrating an excellent understanding of aspects of mathematics.
	b. Breaking down the lesson content into key components	Critically breaking down the lesson content into its fundamental components so that it is easy to understand and applicable
	c. Mathematical structures and connections	Connecting concepts and topics
	d. Procedural knowledge	Having excellent skills in solving math problems
	e. Mathematical problem-solving methods	Using a method (a certain way) in solving mathematical problems
3	*Pedagogical skills concerning the ability to understand concepts*	
	a. Learning objectives	Explaining the learning objectives (general or specific objectives)

b. Gaining and maintaining students' focus	Discussing strategies to help students be actively involved and encourage them to be engaged during learning.
c. Class management	Discussing general rules applied in the classroom

PCK in RME

PCK teachers can apply the principles of RME by learning with real problems. The teachers used open-ended questions to obtain varied answers. In this study, before the PCK was carried out, the teachers were trained first to overcome possible problems in the classroom. One of the guidance activities is designing a learning trajectory. These activities are performed to assist teachers in making learning becomes more directed. With adequate training, the teacher will be more focused on overcoming common problems in the classroom, such as misconceptions.

Research on PCK has been widely conducted in education, such as by Ma'rufi et al. (2018), about teacher PCK in mathematics learning on function limit materials. In addition, Mohd et al. (2021) researched the PCK assessment of Intermediate Mathematics Teachers. Barut et al. (2021) also studied the relationship between PCK in mathematics teachers and the learning achievements of junior high school students. However, no one has researched the PCK of teachers using the RME approach to geometry material. Geometry is an essential material to learn and has become an important component of various aspects of life (Jones, 2002). In addition, students still think geometry material is challenging to learn (Jelatu et al., 2018; Tutkun & Ozturk, 2013). Therefore, teachers need to pay attention and prepare their teaching. They need the necessary knowledge and skills, such as content, general pedagogical, and pedagogical content knowledge, to create effective learning (van Driel et al., 2002). Therefore, this study explores how the teacher's PCK applies the RME approach in geometry.

Methods

The study used descriptive qualitative, describing extensive phenomena using various natural methods (Creswell, 2017). The stages of the study were conducted based on the analysis of Miles and Huberman (2014), consisting of data reduction, data presentation, and conclusion. This study aimed to analyze the PCK of teachers in applying the RME approach to learning geometry. The instruments in this study were observation sheets and interviews. Data were collected through observations and interviews capturing how teachers teach using RME, referring to the teacher's PCK. Researchers also reflected on the learning process to find its weaknesses and strengths to improve the learning next time. In addition, before teaching the lessons, the teacher gets training as many as twice related to the lessons.

This study utilized mathematics learning with RME held in two meetings to see the PCK of teachers. After completing the first lesson, the researchers and teachers reflect on the learning—the reflection aimed to improve the teaching quality for the next class, train the teacher's PCK in applying RME and create more meaningful learning.

The research participants were two elementary school teachers in Aceh province, Indonesia: H and L. RME training was provided for the participants, especially in geometry. The teachers involved in the training carried out by the researchers were six low-class teachers. The subjects of this study were 2 grade 3 teachers who taught geometry material, while the other teachers took part in other RME research. Before the lessons, participants also discussed with researchers to obtain a more targeted learning experience following student characteristics for optimal learning outcomes. This study utilized an observation sheet to observe teachers' PCK. The indicators of PCK teachers in applying the RME approach were adapted from Maher et al. (2015).

Results and Discussion

In this study, the learning took place in two meetings. The first meeting consisted of three activities. Activity 1 explored students' prior knowledge about measuring the area of a plane geometry using concrete objects; activity 2 was to finds\ the area of a plane geometry, and activity 3 examined students' ability to determine the area of a plane geometry. The second meeting had three activities: determining the area of a rectangle using non-standard units, learning trajectories, and student worksheets designed with the theme of energy sources. The material in this study was to determine the area of a plane geometry using non-standard units. The following was a dialogue between teachers H and L with students during the lesson.

Teacher H

Meeting 1

Teacher H : Which is the widest garden?
Students : The fourth garden
Teacher H : And then?
Students : That is the only one
Teacher H : Which is the biggest table? The one which is square or rectangle?
Students : rectangle
Teacher H : Right. Which shape is best to determine the area of the tablecloth?
Students : Square (choir)
Teacher H : Do you know why it cannot be with a circle shape?
Students : Yes, we do. It can have some gaps.
Teacher H : That is true.

Meeting 2

Teacher H : Does anyone know what the standard unit is?
Students : Cm, m, ruler
Teacher H : That's right. kalau satuan tidak baku ada yang tau?
Students : Fingers, hands, arms, ropes, nails
Teacher H : It's true
Students : Miss. How is it? (a student pointed out the student worksheet)?
Teacher H : Make a square shape with post it and draw it on the student worksheet.
Students : What's next?
Teacher H : Draw another rectangular shape.

The dialogue indicated that after learning, students already understand measurements with non-standard units, namely using square and circle shapes to determine the area of squares and rectangles. However, during the lesson, the teacher only asked the final answer of the student's assignment on the student worksheet and did not ask further questions regarding the student's answer.

Teacher L

Meeting 1

Teacher L : What are examples of non-standard units?
Student : Span, foot, span, fathom, cubits
Teacher L : Look at the floor (ceramic). What shape is it?
Student : Square
Teacher L : Yes, one of the flat shapes is a square
Teacher L : Which part of the cabbage garden is the most extensive?
Student : Garden part 4
Teacher L : How to determine it?
Student : Count the number of cauliflowers
Teacher L : Which garden is the smallest?
Students : Part 1, part 2 (student answers vary)
Teacher L : The correct one is part 2
Student : It doesn't cover completely. Do you see a gap?
Teacher L : That's ok. The important thing is that the patches are parallel
Which table is wider (circle plate)?
Student : square, rectangle (student answers vary)
Teacher L : How do you know if a square is wider than a rectangle?
Student : A square table contains 16 plates, while a rectangular table contains 12.
Teacher L : Yes, very good.
Which tablecloth shows a full table without gaps?
Student : Square (answer simultaneously)
Teacher L : Yes, that's right
Which is wider, the table area of the heat energy or with chemical energy?
Student : Heat energy
Teacher L : Why?
Student : There are six tables for the heat energy group, while there are four chemical energy group tables
Teacher L : How do you calculate it?
Student : We multiply it.
Teacher L : How do you multiply it?
Student : 3 × 2
Teacher L : Why don't you just count one by one?
Student : It takes time. It's faster if we multiply it.
Teacher L : Yes, that's right

Meeting 2

Teacher L : What shape is the table?
Student : Rectangle
Teacher L : How many origami papers do you need to make a rectangle?
Students : Eight (answer simultaneously).
Teacher L : How do you calculate it? (students come forward)
Student : 4 × 2
Teacher L : Which are 2 and 4?
Student : (shows that there are two rows with four origami in each row)
Teacher L : Why do you multiply it? Why don't you count one by one?

Student : It takes time. It is faster and easier if we multiply it.
Teacher L : Yes, very good.
How is the length?
Student : 6
Teacher L : How is the width?
Student : 2
Teacher L : Write them in the worksheet.
What is the conclusion from the lesson?
Students : Calculate the area using non-standard units, using origami
Teacher L : How is activity 2?
Student : Count the biggest biscuits.
Teacher L : How is activity 3?
Students : People install ceramics.
Teacher L : Yes, the number of ceramics needed

After learning, the dialogue indicated that the students could understand area measurements using non-standard units. During the learning process, the teacher continuously checks whether students understand or not what is being explained. The teacher always sees the students' progress in answering questions rather than checking the final answer.

Furthermore, the results are presented based on the observational data collected during the study. The following is data on the PCK of teachers H and L in implementing the RME approach. According to Maher et al. (2015), PCK has sixteen indicators from three PCK categories: clearly PCK, the ability to understand concepts concerning pedagogical abilities, and pedagogical skills concerning the ability to understand concepts. The following is an explanation regarding the PCK category and indicators.

Clearly PCK

PCK is a PCK category with eight indicators, namely discussing and using strategies or approaches in teaching mathematical concepts, discussing/directing students to think about concepts or trying to help students to be at a certain level of understanding, discussing and helping students who have misconceptions, identifying the task aspects affecting the cognitive level (C1, C2), explaining and demonstrating concepts using models or illustrations (e.g., props, diagrams), discussing/using learning resources during learning, discussing how the topics taught following the curriculum (based on previous and future grades), and discussing the relationship of the topic/subject with other relevant materials or applying the concept. Teachers H and L have fulfilled 5 of the 8 PCK indicators. H and L have not fulfilled the indicators of discussing and using strategies or approaches in teaching mathematical concepts, identifying the task aspects affecting the cognitive level (C1, C2) and discussing and using learning resources during learning.

The ability to understand concepts concerning pedagogical abilities

The ability to understand concepts concerning pedagogical abilities has five indicators, namely demonstrating an excellent understanding of aspects of mathematics, critically breaking down the lesson content into its fundamental components so that it is easy to understand and applicable, connecting concepts and topics, having excellent skills in solving math problems and using a method (a certain way) in solving mathematical problems. Teachers H and L have fulfilled three of the five indicators of the ability to understand concepts about pedagogical abilities. H and L have not met the indicators of having excellent skills in solving math problems and using a method (a certain way) in solving mathematical problems.

Pedagogical skills concerning the ability to understand concepts

Pedagogical skills concerning the ability to understand concepts have three indicators: explaining the learning objectives (general or specific objectives), discussing strategies to encourage students' engagement in learning, and discussing general rules applied in the classroom. Teachers H and L have fulfilled two of the three indicators of the skills concerning the ability to understand concepts. They could not achieve one of the indicators, namely, discussing strategies to enhance students' engagement in learning. Overall, H and L fulfilled ten of the sixteen PCK indicators. The following is an excerpt from the researcher's interview with teachers H and L.

Teachers H

Researcher : What are the strategies to create more effective learning?
Teachers H : Using props.
Researcher : What is your strategy if students need help understanding after having explanations?
Teachers H : I use the student worksheet to have more directed learning.
Researcher : What do you do if the students still need help understanding seen from the gaps in the student worksheet?
Teachers H : I explain it again to make them understand.
Researcher : What are your efforts if the students have a misconception?
Teachers H : We had no misconceptions during learning.
Researcher : How do you give questions to your students?
Teachers H : The problems are mixed, starting from low, medium, and high levels.
Researcher : Are the teaching aids appropriate?
Teachers H : There are pictures of circles and squares so students can find the concept.
Researcher : How many learning resources do you use in learning?
Teachers H : I use videos and power point slides.
Researcher : Do you search for other learning resources?
Teachers H : Yes, I search for them on the internet.
Researcher : How do you teach the students to follow the curriculum to achieve the learning objectives?
Teachers H : I adjust the learning to the curriculum.
Researcher : How do you learn the curriculum?
Teachers H : I learn it from school.
Researcher : Are there connections between one material and others in learning?
Teachers H : Yes, such as arts, crafts, and Pancasila and citizenship education
Researcher : What aspects of mathematics do you understand in teaching?
Teachers H : I must understand the concept of finding the area of plane geometry.
Researcher : Have you prepared a learning trajectory (core components) with several conjectures, such as possible questions that students will ask during learning activity?
Teachers H : I prepare the student worksheet *to enhance students' understanding of finding an area in squares and rectangular.*

Researcher : How do you explain the correlation of the material to real life?
Teachers H : I explain to my students that mathematics is closely related to real life.
Researcher : What is your strategy if students focus on something other than learning?
Teachers H : I apply the problem-based learning model to solve problems to get a broad concept, check the student's work in each group to determine whether they are working on the student worksheet, and explain the worksheet's steps.
Researcher : Do you have some rules in the classroom?
Teachers H : Yes. The students can have a break earlier if they can finish their tasks. They must present their work and raise their hand to ask and answer questions.

Based on the interview, H has directed the students to think about the concepts, as seen in the early learning activities. H also tried to help students reach a certain level of understanding. H has also directed students to think at the expected level, such as understanding concepts. In learning, H directed students to think by proposing provoking questions. Furthermore, H also helped students with misconceptions, as seen in students' work on worksheets. The worksheet asked students to find the area of 2D shapes, but students were finding the circumference. H also demonstrated concepts using models or illustrations, such as interactive media (Figure 1) for all groups.

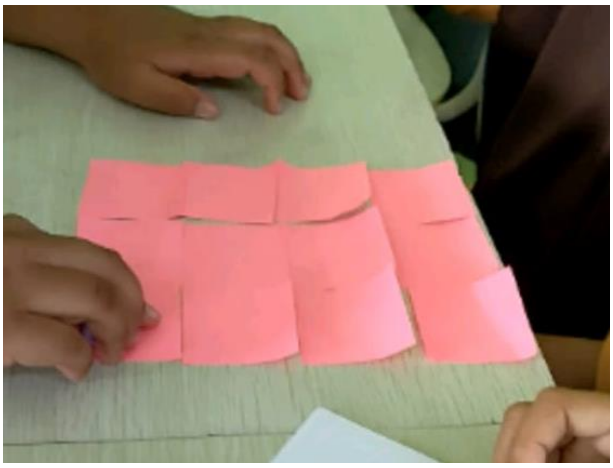

Figure 1. The use of learning media by teacher H

Before the learning, H also discussed the teaching topics with the researchers to align with the 2013 curriculum. Thematic books are used in the curriculum; the learning was thematic-based and related to energy sources. H also discussed with the researchers the learning that was in line with the student's characteristics and the duration of learning. The topic is related to other relevant topics or the implementation of the concept, such as Science lessons regarding energy sources and Art, Culture, and Craft lesson.

H demonstrated an excellent understanding of mathematical aspects. In addition, H elaborated the lesson content into fundamental components to make it easy to understand and apply. Thus, the concepts and topics are related, for example, the concept of area and multiplication as repeated addition.

At the beginning of the lesson, H also explained the learning objectives (general or specific). H discussed the general rules applied in the classroom. However, these rules were not applied as planned due to several factors, such as the time constraint.

H had fulfilled several indicators of teacher PCK. However, H was less focused on applying RME. H emphasized students' answers more than the students' process of obtaining answers. H understands the concept well, but the explanation was too monotonous, leading to less meaningful learning for students. H explained the concept of A=w×l directly.

Nevertheless, H could engage students in learning. H had the initiative to design the learning according to student characteristics. All groups also got equal attention during the learning as H walked around checking all the activities done by students on the worksheet.

Teachers L

Researcher : What are the strategies to create more effective learning?

Teachers L : I use media; learning begins with real problems that are close to students' lives, such as cauliflower gardens which have arbitrary shapes; the problems presented are not directly square or rectangular.

Researcher : What is your strategy if students need help understanding after having explanations?

Teachers L : I give more examples that students can see directly (in the classroom) to encourage students to think.

Researcher : What will you do if students still need help understanding seen from the gaps that appear in the student worksheet*?*

Teachers L : I encourage them to think, such as cutting a unit square to cover and avoid the gaps.

Researcher : What are your efforts if the students have a misconception?

Teachers L : I use media to teach the concepts.

Researcher : How do you give questions to your students?

Teachers L : The problems are mixed, starting from low, medium, and high levels.

Researcher : Are the teaching aids appropriate?

Teachers L : The teaching aids ease students to reach a more formal level of knowledge.

Researcher : How many learning resources do you use in learning?

Teachers L : It depends on the materials and needs. I usually use those around the school; for example, there are blackboards, ceramics, wall magazines, and so on in plane geometry materials.

Researcher : Do you search for other learning resources?

Teachers L : Yes, I usually search for them online, ask co-workers, and discuss them.

Researcher : How do you teach the students to follow the curriculum to achieve the learning objectives?

Teachers L : I adjusted the materials, concepts, textbooks, evaluation, and needs of the teaching material.

Researcher : How do you learn the curriculum?

Teachers L : I had guidance from school about how to implement it in the classroom. I adjusted and learned the curriculum.

Researcher : Are there connections between one material and others in learning?

Teachers L : Yes, there are. For example, in area measurement topics, there are materials for plane geometry, multiplication, and other learning materials (intertwin).
Researcher : What aspects of mathematics do you understand in teaching?
Teachers L : Multiplication, subtraction, division, etc.
Researcher : Have you prepared a learning trajectory (core components) with several conjectures, such as possible questions that students will ask during the learning activity?
Teachers L : Yes, I have some notes.
Researcher : How do you explain the relationship between the material and real life?
Teachers L : Explain the benefits of what is learned and its application in life
Researcher : What is your strategy if students focus on something other than learning?
Teachers L : Having ice breaking and game.
Researcher : Do you have some rules in the classroom?
Teachers L : Yes, there are. If the students want to answer or ask questions, they have to raise their hand, or if they want to go to the restroom, they need to ask for permission. I set the time for them to complete their assignments. Also, rewards include the neatest group, the fastest ready to get a star.

Based on the interview, L tried to support students to be at a certain level of understanding by directing students to think about the mathematical concepts to be achieved. Students were directed to think to the expected level, and L directed students to think by proposing provoking questions and providing examples of the implementation of mathematics in everyday life for students to imagine. L has also helped students to overcome misconceptions. L used interactive media (Figure 2) such as teaching aids and prepared additional media to help students understand the lesson better and think critically.

Figure 2. The use of learning media by teacher L

Before the learning, L discussed with researchers about the learning, such as choosing topics following the 2013 curriculum, thematic-based learning, and related to energy sources. The topic is related to other relevant topics or the implementation of the concept, such as Science lessons regarding energy sources and Art, Culture, and Craft lesson.

L had an excellent understanding of mathematical aspects. In addition, L elaborated the lesson content into fundamental components for students to easily understand. This made the concepts and learning topics interrelated, such as the concepts of area and multiplication. Before starting the activity, an explanation of the learning objectives (general or specific objectives) and the applicable class rules. However, the run did not run smoothly.

L had met several indicators of teacher PCK and applied the RME approach well. In the teaching and learning process, L followed the student's flow of thinking in determining a concept so that learning was more meaningful. The understanding of the concept was good, and learning went as expected. L's explanation was not directly on the concept of $A=w\times l$; instead, L tried to transfer the meaning of the concept, such as explaining that area is the number of unit squares covering the shapes without gaps.

L has applied the RME approach in learning mathematics several times to achieve the learning objectives. L was concerned about the duration required to complete activities in the worksheet so that students could complete these activities. All group answers were checked one by one to ensure no incorrect answers. L applied the learning trajectories sequentially. Thus no stage was missed in applying the RME approach.

H and L fulfill ten of the sixteen PCK indicators. The indicators achieved by H and L are the same indicator. Thus, it means that teachers H and L have the same PCK. Not all PCK indicators were achieved by the teachers. However, the teacher's PCK could be better. Some indicators do not appear due to the incompatibility of the characteristics of the selected material.

The teachers' confidence during the teaching and learning process also sharply increased, so learning was not rigid and monotonous. Class management was excellent because the teacher prepared the lesson well during guidance and taught seriously to obtain optimal results.

During the learning implementation of RME, the teacher was very enthusiastic. All students were engaged in learning, and student engagement indicates that the teacher's PCK steadily improved. The assistance by researchers in implementing RME benefited the teacher.

During the research, in addition to guiding teachers H and L, the observers also reflected on the learning carried out to discuss the shortcomings of the teaching and learning, enhancing the teacher's PCK in learning for the next meeting. The teachers also reflect on the learning conducted.

H believed that RME met the student characteristics, such as using varied learning media. However, H was confused about applying some RME principles during the lesson. On the other hand, in the teaching and learning implementation of RME, L was better. L was able to engage the students in expressing opinions. The teaching props used during the teaching and learning process were very interesting.

The differences between RME implementation in mathematics between H and L are due to several factors. One of them is an experience in applying the RME approach in learning. It was H's first experience implementing RME, while L has implemented the approach and some related training before. Therefore, the guidance or assistance provided for teaching preparation is crucial to support better PCK.

The PCK of teachers in implementing RME has fulfilled ten of the sixteen PCK indicators to several factors, such as guidance before the lesson. This guidance impacts a more directed teaching and learning process, making learning more meaningful. These findings align with previous studies that teacher PCK positively affects the quality of learning and student progress (Baumert et al., 2010; Coe et al., 2014). Therefore, it is important to stimulate teachers' PCK during teaching guidance (Sorge et al., 2017; Kind, 2017; Nilsson & Loughran, 2012).

PCK has an important role in the professional development of teachers (Smith, 2007). So, teaching guidance is necessary for improving PCK. Teachers' PCK is also considered a prerequisite for teaching activities (Baumert et al., 2010; Hill et al., 2008). Teachers' PCK is directly related to mathematics teaching competence (NAE & NRC, 2014). Hammack and Ivey (2017) found that teaching competence is closely related to teachers' PCK. Thus, a positive relationship between teaching competence and PCK is confirmed (Richardson et al., 2018; Thomson et al., 2017).

In addition, this study indicates that learning with RME not only affects the teachers' PCK but also significantly improves student engagement in learning. This finding agrees with Afthina's research (2017), revealing that learning mathematics with RME improves learning outcomes. It is also possible due to the use of real context. A similar finding was reported by Meletiou-Mavrotheris and Paparistodemou (2015) that using contexts familiar to students benefits the implementation of the learning process. Research by Bray and Tangney (2016) also showed that learning mathematics with RME can increase students' self-confidence and engagement in following lessons, as well as reduce students' learning anxiety about learning.

Conclusion

The teacher's PCK in applying the RME in geometry fulfilled ten of the sixteen PCK indicators after the guidance of the learning. The application of RME in teaching and learning makes this learning more meaningful. Students can also relate mathematics learning to everyday life. The application of RME in learning enhance teachers' PCK. Since teachers at the elementary school teach almost all subjects, it is hoped that their PCK can be upgraded to a higher quality. It is challenging to determine a teacher's PCK; therefore, a different measurement method and a long-term investigation are needed to achieve more accurate results. In addition, teachers need to be aware of their level of PCK to trigger self-improvement.

Acknowledgment

We would like to thank Univeritas Syiah Kuala for funding this research project (Grant Number: 303/UN11.2.1/PT.01.03/PNBP/2022).

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