

Mapping students' thinking structure in solving PISA-like problems: Errors and defragmentation

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Abstrak Kesalahan dalam pemecahan masalah menunjukkan bahwa ada bagian struktur berpikir yang bermasalah, baik karena tidak teratur, terputus atau mengalami lubang kognitif. Penelitian ini mengeksplorasi kesalahan struktur berpikir siswa dalam menyelesaikan masalah PISA dan mengusulkan upaya defragmentasi untuk mengatasi masalah tersebut. Partisipan penelitian terdiri dari tiga siswa kelas X SMA dengan kemampuan berbeda-beda, yaitu tinggi, sedang, dan rendah, yang dipilih melalui purposive sampling. Pengumpulan data dilakukan dengan tes dan wawancara semi terstruktur, dengan triangulasi digunakan untuk menjamin keabsahan data. Analisisnya meliputi penyajian hasil tes dan transkrip wawancara, pemeriksaan lembar dan transkrip secara menyeluruh, kategorisasi kesalahan struktur berpikir, dan penjabaran upaya defragmentasi berdasarkan tahapan pemecahan masalah. Temuan penelitian mengidentifikasi variasi kesalahan struktur berpikir dan strategi defragmentasi di antara para peserta. Penelitian ini menunjukkan pentingnya pengajaran dengan fokus pada penguatan pemahaman konsep prasyarat, peningkatan keterampilan literasi membaca, dan diversifikasi pendekatan defragmentasi, khususnya dalam penerapan strategi scaffolding-explaining, untuk meningkatkan kualitas pemecahan masalah matematika siswa.

Kata kunci *Struktur berpikir, PISA, Error, Defragmenting*

Abstract Errors in problem solving indicate that there is a part of the thinking structure that is problematic, either because it is disorganized, disconnected or experiencing cognitive holes. This research explores students' thinking structure errors in solving PISA problems and proposes defragmentation efforts to address these issues. The study participants consist of three high school grade X students with varying abilities high, medium, and low selected through purposive sampling. Data collection involves tests and semi-structured interviews, with triangulation used to ensure data validity. The analysis includes presenting test results and interview transcripts, thorough examination of sheets and transcripts, categorization of thinking structure errors, and elaboration of defragmentation efforts based on problem-solving stages. The study findings identify variations in thinking structure errors and defragmentation strategies among participants. This research shows the importance of teaching with a focus on strengthening understanding of prerequisite concepts, improving reading literacy skills, and diversifying defragmentation approaches, especially in applying scaffolding-explaining strategies, to improve the quality of students' mathematical problem solving.

Keywords *Thinking structure, PISA, Error, Defragmenting*

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Introduction

PISA Mathematics aims to reveal the extent to which students are able to overcome the challenges of solving mathematical problems in real-world contexts. The problems presented are designed to measure students' readiness to face real life challenges, by emphasizing the application of knowledge and critical thinking (OECD, 2011) so that the question format does not only require the ability to apply concepts, but also how these concepts can be applied in various situations (Kurniati et al., 2016). Students' inability to solve PISA questions shows that there is a problem in their thinking process, whether due to understanding related concepts, application or other factors that could possibly influence (Abadi et al., 2017).

In research on students' abilities in solving PISA questions, (Lutfianto et al., 2013) revealed that the errors most often experienced by students in working on PISA questions lie in getting mathematical work results, then the answers cannot be interpreted in the desired problem situation/context. In (Sa'adah et al., 2019) the findings revealed that students' failure in solving problems was mostly caused by misunderstanding the problem. This can happen when students do not comprehensively understand the information provided in the question. Apart from that (Fazzilah et al., 2020) in their findings showed something different, namely that the errors they got the most were errors in transforming questions, as many as 55% of students' errors were caused by errors in using the correct question formula. served. For example, in a geometry question in PISA, students may misinterpret the question as being about finding the area of a triangle when the question is actually about the perimeter. (Fadhilah & Rosyidi, 2020) stated that students' mistakes will vary depending on the content of the material in the PISA questions given. The level of students' mathematical problem solving abilities influences the types of errors experienced (Djadir, Awi, 2018). Students with a good understanding of problem solving will be able to solve PISA questions correctly (Mardhiah et al., 2021).

In the existing research literature, attention to the relationship between each step of problem solving is actually interrelated in constructing the structure of thinking in the stages of problem solving. Even though in the problem solving process it is not only PISA problems, the process is interrelated at every stage of thinking, linking various cognitive parts and other knowledge to one another, these parts connected to form a series of thinking structures in solving problems (Subanji, 2015). So that is why the difference in content and the level of student knowledge will greatly impact the student's process of solving PISA problems. In this case, there is a structure of problem-solving thinking in the form of a series of cognitive structures built during the problem-solving process (Kumalasari et al., 2016). Therefore, to explore students' problem-solving errors in solving PISA problems, specifically it can be done by mapping the thinking structures experienced by students while working on problems. This can be done because in every problem solving error, it shows that there are parts of the cognitive structure that are problematic, either because they are not organized, not connected or experiencing cognitive holes. (Subanji, 2016; Kumalasari et al., 2016). Through this cognitive mapping, errors in the structure of thinking will be identified and can then be corrected through structuring interventions, known as mathematical *defragmenting* (Subanji, 2015).

Based on the description above, this research aims to describe students' thinking structure errors in solving PISA questions, identify and increase efforts through providing defragmenting interventions. Through findings from previous research highlighting differences

in error types, it becomes clear that the content and level of students' knowledge greatly influences the problem-solving process. By recognizing the relevance of cognitive structures in problem solving, this research takes a new approach by proposing a cognitive mapping method based on Polya theory to identify and overcome errors systematically. The main goal is to pave the way for an effective intervention, known as mathematical defragmentation, to improve students' problem-solving abilities in the context of PISA questions.

Theoretical review

Errors in Problem Solving

Mathematical problem solving has a central role in the learning process, helping to stimulate the development of critical thinking skills, application of mathematical concepts in real life contexts, increasing problem solving abilities, and strengthening concept connection skills (Subanji, 2015a). These abilities are not limited to the classroom context, but extend into everyday life, giving individuals the ability to face challenges with a problem-solving mindset (Polya, 1978). The essence of problem solving is an attempt to find a solution to a difficulty, aimed at achieving a goal that cannot always be achieved directly. Something is considered a problem when it poses a challenge and cannot be overcome in a structured and correct way by someone (Roebyanto & Harmini, 2016). A question can become problematic when a person does not have a specific rule or law that can be used directly to find the answer. Therefore, problem solving is not only related to individual knowledge about the substance of the problem, but also includes understanding the core of the problem, the steps in solving the problem, as well as the rules or formulas needed to solve the problem.

Errors in problem solving indicate that there are parts of the cognitive structure that are problematic, either because they are unorganized, unconnected or experiencing cognitive holes. (Kumalasari et al., 2016). Often the problems faced by students are more complex than their cognitive structure. This requires students to be able to decompose these complex problems into simple information so that they can be resolved. From the completion of each part, a hook is made for each component of the settlement so as to form an overall solution which is called a concept connection (Subanji, 2015a). Often students do not know how to start solving problems, how to relate one concept to another, and how to relate one procedure to another to solve a problem.

To overcome errors in problem solving, it is necessary to trace from the study of students' construction processes in solving problems. By knowing the construction process of students' problem solving in mathematics, this can help in detecting students' mathematical connection errors that students should have built. Therefore the construction process of students in solving problems must be a major concern for educators.

Mapping Structure of Problem Solving Thinking

The thinking process of students is one of the most important factors in the success of students in receiving learning. This is because in the thought process, it will involve working cognitive structures, where the cognitive units of the structure work together with other related ideas at the same time (Subanji, 2016). One part of the activity of the thinking process is problem solving (Estes, 2014). In the process of solving problems linking a series of thinking structures from various cognitive parts and other knowledge to one another in solving

problems (Trevor R. Anderson, 2008). The problem-solving process requires building connections between the stages of problem-solving, as an effort to find solutions based on the knowledge possessed (Tasni & Susanti, 2017). The series occurs during the problem-solving process (Kumalasari et al., 2016). Success in solving problems shows that the cognitive structure are sufficient and connected to each other correctly, conversely the occurrence of errors in problem solving indicates that there are parts of the cognitive structure that are problematic, either because they are disorganized, disconnected or experience cognitive holes (Subanji, 2016; Kumalasari et al., 2016).

To explore the structure of problem solving thinking can be done through cognitive mapping (Wahab, 2022). One well-known cognitive form is a concept map. a technique that has been developed over a period of time and through application has demonstrated its use for operational researchers working to represent structures across a variety of tasks (Ackermann et al., 2004). Cognitive mapping is the process of visualization and graphical representation of a person's thoughts and knowledge. It involves the use of diagrams, graphs, or other visual models to illustrate the relationships between concepts or information that one has in mind (Siau & Tan, 2017). Cognitive mapping can help in understanding the structure of information, relationships between ideas, and assist in decision making or problem solving. In an educational context, cognitive mapping is often used to help students organize and understand lesson material more thoroughly (Budhwar, 1996). Ackermann further stated that cognitive mapping can be used in various ways, including being used to solve problem solving both individually and as a group, this technique can make it easier to identify problems and also create a problem structure to be solved. In addition, through cognitive mapping it will be able to help formulate difficulties experienced by students and determine appropriate steps for assistance (Abadi et al., 2017).

Cognitive mapping in problem solving, requires a design that describes the flow of thinking or steps for solving a problem that is coherent and clear. A clear and coherent description of the flow of thinking or problem solving steps can be viewed from the strategy used because it is an important part of problem solving. Therefore (Polya, 1978) devised a heuristic strategy to guide problem solving in finding solutions to problems, namely 4 general steps, namely understanding the problem, devising a plan, carrying out the plan and looking back.

In this study, problem-solving thinking structure mapping is carried out through cognitive mapping based on polya theory, namely in the form of a representation technique for thinking structure process models expressed through cognitive search for problems. The solution is in the form of a structured and systematic pattern outlined in graphic format adapted to the Polya problem solving steps. Through polia-based cognitive mapping, researchers will be able to better formulate the difficulties experienced by students in a coherent and clear (Buhaerah, 2022). This is because Polya's approach in the context of mathematical problem solving provides a systematic framework for identifying, analyzing and overcoming difficulties in the problem solving process. Thus, this approach helps organize understanding of the obstacles faced by students so that they can be presented in a more structured and easy to understand manner.

Defragmenting Thinking Structures

In the world of learning, the term defragmenting was first used by (Subanji, 2015a) which refers to a form of activity for organizing and improving the structure of students' thinking (Kumalasari et al., 2016). Defragmenting is the process of rearranging students' thoughts that have not been organized so that it causes students to experience errors in solving problems regularly so as to gain a deep understanding and be able to solve problems properly (Subanji, 2016) Defragmenting can also be interpreted as cognitive restructuring in individuals (Zubaeda, 2020).

Defragmenting is usually done if the student's thinking structure (a person has appeared or has been formed but errors still occur in solving a given problem (KA Wibawa, 2016). Because only with a complete thinking structure, students can solve the problems they encounter (Haryanti, 2018) This intervention can be carried out in two stages, namely the first stage identifies errors in the thinking structure, and the second stage structuring the thinking structure to be correct Both stages are carried out through several interventions that are adjusted to the problems in the thinking structure experienced, these interventions include: scaffolding, disequilibrium, and cognitive conflict (Sasongko et al., 2018).

Scaffolding is an attempt to provide assistance in the form of questions, instructions, reminders, directions, or encouragement to students when they experience errors in solving problems (Kumalasari et al., 2016). According to (Anghileri, 2006) this intervention can be classified into three levels, namely:

1. Level 1 (environmental provisions) is in the form of assistance not directly on math content but on the learning environment.
2. Level 2 (explaining, reviewing, and restructuring) the assistance provided focuses on math content. The assistance provided can be in the form of questions to direct students to identify all aspects related to the problem, to encourage students to convey ideas and their justifications, as well as to emphasize the discussions being carried out.
3. Level 3 (making connections and generating conceptual discourse) is an extension of justification at level 2 which results in the emergence of many concepts.

In this study applying scaffolding level 2, because the process of solving PISA problems emphasizes concepts related to mathematical reasoning, so that student difficulties will require assistance interventions in the form of instructions, encouragement, warnings, giving examples, and other actions that allow students to involve their awareness of thinking about the processes and results of a mathematical problem.

Disequilibrium is defined as the fruit of intervention in the form of assistance to students to reflect on the work done by comparing it to existing scientific concepts. Disequilibrium is done when students are unable to connect knowledge to produce solutions to problems (Subanji, 2016). This intervention is characterized by the appearance of students having doubts about cognitive knowledge which is known to be due to an imbalance in the state of mind when asked a question (Damayanti et al., 2020). Disequilibrium is carried out, by asking interventions in the form of questions, so that students experience confusion which indicates an imbalance between accommodation and assimilation (Kumalasari et al., 2016).

Cognitive conflict can be interpreted as the assistance given to students when they are faced with contradictions or inconsistencies when expressing ideas. (Subanji, 2015a). Cognitive conflict can change beliefs in learning mathematics (Rolka & Liljedahl, 2007). Cognitive conflict is done by asking questions and examples that are different from the

answers given by the participants so that the participants have conflicts in their minds and they rethink the answers.

The three interventions are adjusted to accelerate the student learning process towards discovering more scientific concepts (Subanji, 2016).

Methods

This research seeks to reveal in-depth understanding, detailed descriptions, and explore new aspects of the analysis of students' problem solving errors in solving PISA questions, as well as efforts to improve them by carrying out defragmentation. Based on the purpose of the research, according to (Labuschagne, 2015) this research is classified as research with a qualitative approach with a descriptive-explorative setting.

Participant

The research participants were a class of 35 students involved in sample selection. Based on the results of exams and interviews, researchers identified three groups of participants based on the level of errors in answering questions: Group 1 (Students who answered all questions correctly). Group 2 (Students who were only able to answer two questions correctly). Group 3 (Students who were only able to answer one question correctly). After carrying out the analysis, the researcher selected three research participants representing each ability group: S1 (high level), Masters (medium level), and S3 (low level) with the additional qualification of students with the best communication skills, the aim was to reveal the thinking process. going well. These participants were considered to represent variations in the abilities of class The data obtained from these three groups can then be used for further analysis related to the research objectives.

Instrument

There are two kinds of instruments used in this study including the main instruments and auxiliary instruments. The main instrument is the researcher himself, who collects research data. While the auxiliary instruments are tests and interview guidelines.

The test instrument used was a written test in the form of 3 PISA math literacy problems in 2012 which were developed by researchers through a revision adjustment process until the test stage was declared valid by 2 mathematics education experts with validation criteria carried out including: (1) The problem items are in accordance with the research concept, namely problem solving, (2) The problem items are clearly formulated, (3) Communicative sentences and standard language, and (4) Sentence structures do not have multiple interpretations. [Table 1](#) shows several examples of test instruments used.

Table 1. Example of test instrument item samples

A genuine problem	Problem after adjustment												
<p>Description: Understand the relationship between the language of a problem and the symbolic and formal language needed to represent it mathematically</p> <p>1. During the following weeks, Ivan deleted some photos and music, but also added new files of photos and music. The new disk status is shown in the table below:</p> <table border="1" style="margin-left: 20px;"> <tr> <td>Music</td> <td>550MB</td> </tr> <tr> <td>Photo</td> <td>338MB</td> </tr> <tr> <td>Free space</td> <td>112MB</td> </tr> </table> <p>His brother gives him a new memory stick with a capacity of 2GB (2000 MB), which is totally empty. Ivan transfers the contents of his old memory stick onto the new one. Which one of the following graphs represents the new memory stick's disk status? Circle A, B, C or D.</p> <div style="display: flex; flex-wrap: wrap;"> <div style="width: 50%; text-align: center;"> <p>A</p> </div> <div style="width: 50%; text-align: center;"> <p>B</p> </div> <div style="width: 50%; text-align: center;"> <p>C</p> </div> <div style="width: 50%; text-align: center;"> <p>D</p> </div> </div>	Music	550MB	Photo	338MB	Free space	112MB	<p>Indicator: Understanding the relationship between the problem language, the symbolic and formal language needed to represent it mathematically.</p> <p>1. Ivan has a memory stick that stores music and photos. The memory stick has a capacity of 1 GB (1000 MB). The status of the disk is shown in the table below:</p> <table border="1" style="margin-left: 20px;"> <tr> <td>Music</td> <td>550MB</td> </tr> <tr> <td>Photo</td> <td>338MB</td> </tr> <tr> <td>Free space</td> <td>112MB</td> </tr> </table> <p>Then his brother gave him a new memory stick with a capacity of 2 GB (2000 MB), which was completely empty. Ivan transferred the contents of his old memory stick to the new memory stick. Which of the following graphs shows the status of the new memory stick disk</p> <div style="display: flex; flex-wrap: wrap;"> <div style="width: 50%; text-align: center;"> <p>A</p> </div> <div style="width: 50%; text-align: center;"> <p>B</p> </div> <div style="width: 50%; text-align: center;"> <p>C</p> </div> <div style="width: 50%; text-align: center;"> <p>D</p> </div> </div>	Music	550MB	Photo	338MB	Free space	112MB
Music	550MB												
Photo	338MB												
Free space	112MB												
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Photo	338MB												
Free space	112MB												

The interview instrument developed by the researcher was basic questions to uncover descriptions of students' thinking structure errors in solving problems. However, before the research instrument was used, corrections were made to the stage where the instrument was declared valid by a mathematics education expert. The validation criteria used include: (1) The suitability of the interview questions with the interview objectives (2) The questions used are appropriate for understanding the student's problem solving framework, (3) The interview questions are easy to understand (4) The language used does not contain double meanings (5) The purpose of the question is formulated briefly and clearly.

Table 2 below shows examples of questions on each focus in the interview guide that have been validated with these criteria.

Table 2. Sample questions each focus in the interview guidelines

Problem Solving Based on Polya Theory	Question	Description of the expected answer
<i>Understanding the Problem</i>	What do you know about this problem?	Students explain substantially information from known problems.
	What is asked of this problem?	Students explain the main points of the problem from the problem understood.
<i>Devising a Plan</i>	How do you solve this problem?	Students reveal a solution plan that is in accordance with the problem understood.
	What steps will you take to solve the problem?	Students explain the stages carried out in the plan made.
<i>Carrying Out the Plan</i>	After getting the math formula / solution plan, can you solve the problem so that you get the result?	Students explain in detail how the plan process is made to get answers to problems
<i>Looking Back</i>	Are you sure your answer is correct? If sure How do you check your answer?	Students express answers based on the solutions that have been made.

Table 2 shows some examples of questions used to gather information from participants. Because the interviews conducted in this study were semi-structured interviews, the sample questions could be developed according to the participants' answers during the interview.

Data collection and analysis

Data collection was done with think aloud to express ideas that were used by using verbal or spoken sentences so that the data obtained were spoken words and written words through student answers (Lundgrén-Laine & Salanterä, 2010). The written test was carried out in one meeting with a duration of 60 minutes to capture the main participants in the study. Furthermore, semi-structured interviews were conducted in two stages. The first stage was carried out for each student to reveal the actual thought process in solving the problem. To obtain valid data, the results triangulation technique is carried out by comparing data or information from tests and interviews (Brender, 2006). From this screening, 3 students were selected with high, medium and low error rates who had good communication skills. Then a second stage of interviews was carried out with 3 selected participants to confirm the data from the first interview (time triangulation) and a defragmentation process was carried out to correct the errors made.

The data analysis technique used in this study adopted the model (Matthew B. Miles, 1984), including (1) Presenting data on test results and interview transcripts from recorded interviews between researchers and research participants; (2) reviewing the test sheets and interview transcripts thoroughly; (3) the categorization of types of thinking structure errors and defragmenting efforts given to students based on the stages of problem solving learned through data reduction activities; (4) Presenting data describing students' thinking structure errors and defragmenting efforts given (5) drawing conclusions.

The research process flow is presented in Figure 1 below.

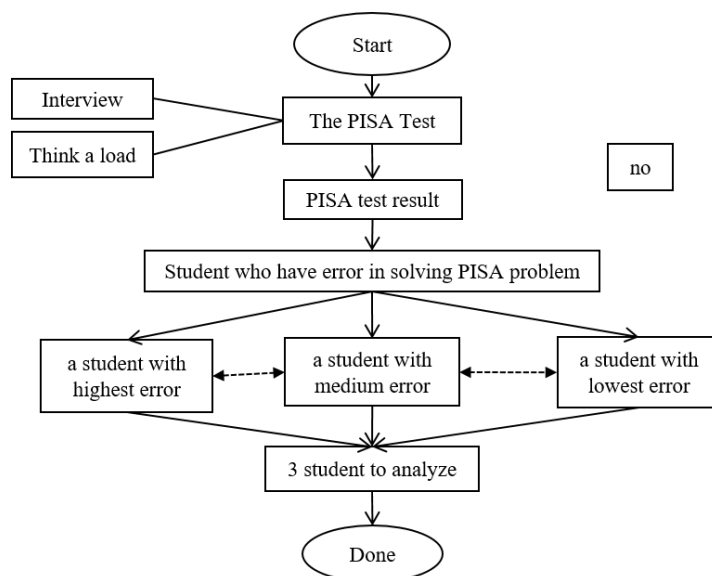


Figure 1 Research Process

Findings

The following are the results of the analysis of the description of the structure of problem solving thinking in participants who have problem solving errors.

The case of S2

Based on the results of the review of written answers and the interview process conducted to uncover errors in the participant's thinking structure, it was found that the Masters succeeded in answering 2 problems correctly, namely problems number 1 and 3. Masters was able to explain the problem, stages, to conclusions based on understanding of mathematical rules and concepts. which is good, this shows that the structure of S2's thinking on these two problems is well organized, so the researcher does not provide defragmenting interventions in solving these two problems. Meanwhile, the error identification results actually occurred in question number 2 when the correct answer shown on the answer sheet did not correctly represent the understanding of the geometric concept of triangles displayed by the question.

Problem Number 2

In this problem, the specific indicator of the requested PISA problem is the ability to use the Pythagorean theorem in a real geometric context. This problem is different from the other 2 problems because in solving it, participants are asked to identify the triangle concept rules needed to apply the Pythagorean theorem rules to the given case. S2's inability to answer problem number 2 correctly indicates that there was an error in understanding and completing the concept. The defragmenting intervention was given to problem number two in the problem, through the following review results.

<p>2. Dik: $y = 150$ m $x = 150$ m Dit: $r = \dots ?$ Penye: $r^2 = y^2 + x^2$ $r^2 = 150^2 + 150^2$ $r^2 = 22.500 + 22.500$ $45.000 = r^2$ $r = \sqrt{45.000}$ $r = 212$</p> <p>Kesimpulan: b. 212 m</p>	<p>Translation Given : $y = 150$ m $x = 150$ m Asked : $r = \dots ?$ Answer : $y^2 + x^2 = r^2$ $150^2 + 150^2 = r^2$ $22.500 + 22.500 = r^2$ $45.000 = r^2$ $r = \sqrt{45.000}$ $r = 212$ Conclusion : b.212 m</p>
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Figure 2. S2's written answer to problem number 2

Based on the written answers in Figure 2, it can be seen that when working on problem number 2, participant S2 directly wrote down the information that was known and asked in the problem by eg in the form of notation. If observed through written answers, it appears that answer S2 has obtained the correct conclusion. However, the interview process that was carried out managed to reveal, there was an error in S2 in understanding problem solving.

The following is an excerpt from an interview that describes the mistakes S2 experienced in problem number 2.

Q : For Number 2, try to explain what you know from the problem?

S2 : First, the length and width are 150 meters each

Q : Why can you say that the sides of this triangle are 150 m each, the lengths of the two sides are not shown in the drawing.

S2 : Because the image in this problem is a right triangle, so automatically if the height is 150 meters, the length of the base must be 150 meters.

Q : Are all right triangles like this?

S2 : Yes sis (showing a very confident expression).

Q : Why are you sure that all the base lengths and heights of a right triangle are always the same length?

S2 : Usually like that sis.

From the excerpt of the dialogue above, it can be seen that even though the final answer made by S2 is correct, however, there seems to be an error in thinking that builds the solution made. S2 is so sure that all the base lengths and heights of a right triangle always have the same length. This answer arises because of unconsciousness that arises because of habit. This perception arises because of the habit of finding similar forms of problems. Perceptions like this have been mentioned by (Vinner, 1997) as a pseudo-state of thinking, namely an understanding that arises from a situation that is not really born as a result of thinking. S2 occurs when calculating the geometry of the sides of a triangle. The main cause of pseudo thinking S2 is due to the habit of working on problems that are similar to the problems given but not accompanied by a good understanding of the concept. According to (Anggraini et al.,

2018) errors that occur due to pseudo thinking need to get attention, because if left unchecked it will have an impact on further mathematical knowledge.

The following is a description of the cognitive mapping of errors in the thinking structure of S2 in problem number 2 which is presented in Figure 3 below.

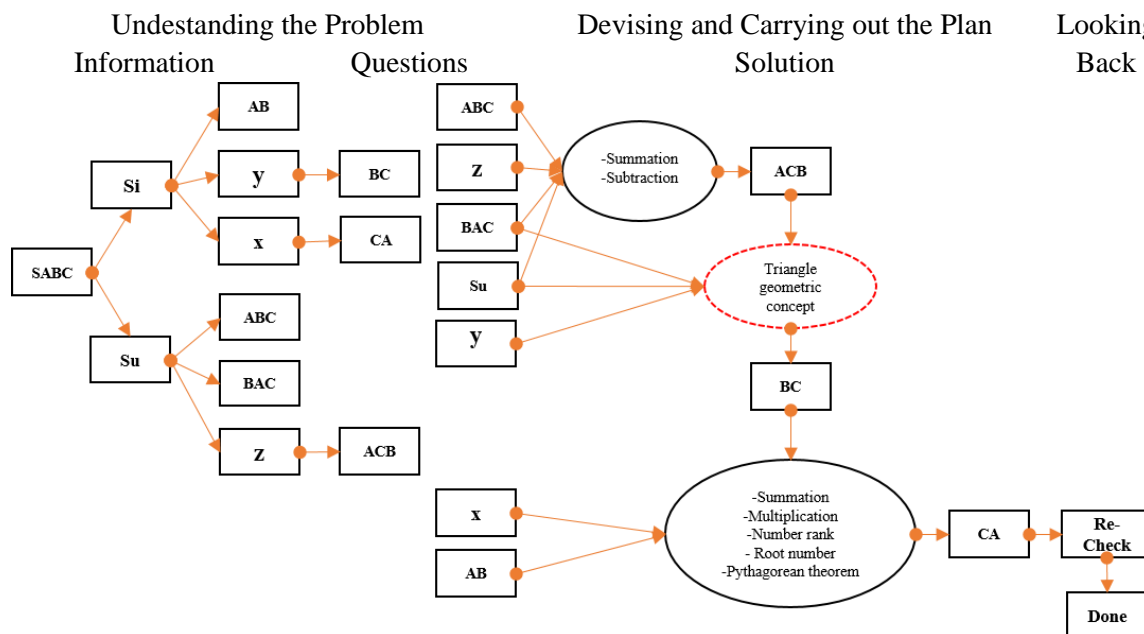


Figure 3 . Mapping S2's thinking structure errors in problem 2

From the results of the mapping of the thinking structure in Figure 3, it shows that S2 is able to understand each component of the information and problems presented by the problem, as well as in the Devising a plan process, S2 is able to arrange steps for solving it until it succeeds in finding AC. However, in reviewing the carrying out the plan components, S2's success in finding AC turned out to be purely coincidental. This is shown in the process of determining BC is not done by applying the concept of geometric triangles but relying on pseudo-non-concept thinking from the experience of practicing similar problems.

In accordance with the S2 error in solving problem number 2, the initial step taken by the researcher in this defragmenting process was to intervene in understanding the geometric concept of a triangle which experienced an error in thinking in determining BC.

The following is an excerpt of the defragmenting interview given by the researcher to S2 to improve understanding and overcome his pseudo-thinking:

Q : Take a look at the triangle that you drew. (Cognitive Conflict)

S1 : (Thinks for a moment, then shows a smiling expression as if he has realized his misunderstanding) Em..

Q : This is a right triangle right?

S1 : yes sis.

P : (While pointing at the triangle picture) If here the length is 150 meters, is this length 150 meters high too? Cognitive Conflict)

S1 : Not the same sis.

Q : Does this mean that the understanding was right or wrong?

S1 : Wrong sis (Equilibration).

From the dialogue excerpt above, after the researcher defragged S2's thinking structure with cognitive conflict intervention in the form of giving similar examples to refute S2's understanding by giving pictures of right triangles which have different side lengths. S2 seems to easily realize the knowledge that has been experienced before is not appropriate in determining BC. This can be seen clearly when the researcher provides a de-equilibration intervention in the form of re-questioning the status of the previous wrong understanding, S2 confidently states that the understanding that was conveyed before is not quite right. This condition according to (Subanji, 2015) above shows the occurrence of equilibrium, namely the existence of a balance of thinking so that the correct understanding occurs. However, to follow up on the improvement of S2's cognitive understanding structure, researchers carried out further defragmenting interventions.

The following is an excerpt of a defragmenting interview given by the researcher to Masters to rearrange previous misunderstandings.

Q : What do you think is the difference between these two right-angled triangles apart from the sides? (Scaffolding-Restructuring)

S2 : Width in degrees

P : (While pointing at the corner of the triangle) Well, that's right, the size of the angle is different. According to S2, how big is this angle?

S2 : 45 °sis

Q : Why do you call this angle a magnitude of 45 °(Scaffolding-Restructuring).

S2 : Eee.(Looks confused face)

Q : Is it because the size of the image is the same?

S2 : Yes sis.

P : Yes indeed, but let me explain. So whatever the shape of the triangle is, if the angles are added together, the total will be 180 degrees. (While pointing to the problem triangle picture) So if you know that the angle measure of this triangle is 90 °, and this is 45 °, then how big is this one angle? (Scaffolding-Explaining)

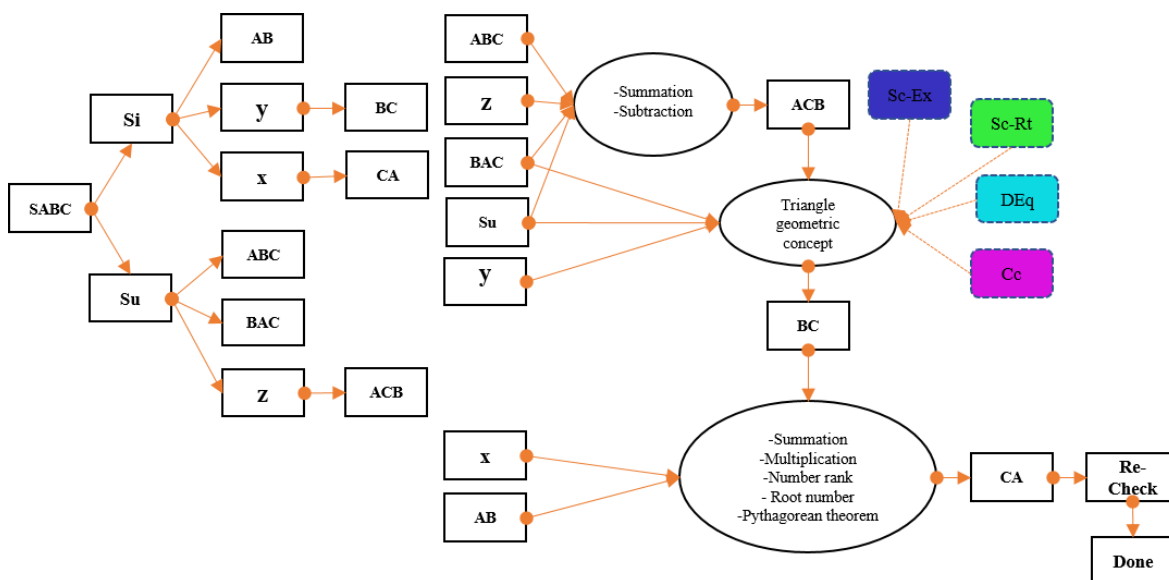
S2 : 45 °sis.

From the dialogue above, it can be seen that the defragmenting intervention carried out by the researcher succeeded in forming a new cognitive understanding of S2 which was previously formed due to the pseudo-thinking experienced. Through scaffolding-restructuring and scaffolding explaining in the form of an explanation of the postulates of the number of degrees of flat angles of the Euclidean geometric triangle, it is shown that S2 is able to correctly determine ABC and ACB.

By knowing that ABC and ACB are angles of 45°, S2 is able to correctly identify SABC so that through this understanding, S2 can understand that if AB is 150 meters long, it can be

ascertained that BC is also 150 meters. Departing from this information, S2 was able to easily determine CA by applying the Pythagorean formula used previously.

The s2 defragmenting intervention for problem number 2 is presented in Figure 4 below.



Description

- Sc-Ex : Scaffolding-Explaining
- Sc-Rt : Scaffolding-Restructuring
- Cc : Cognitive Conflict
- DEq : Disequilibrium

Figure 4 . Defragmenting S2 on issue 2

The case of S3

Based on the results of the review of written answers and the interview process conducted to uncover errors in the participant’s thinking structure, it can be seen that the S3 only succeeded in answering 1 problem correctly, namely problem number 2. On this problem, S2 was able to explain the problem, stages, to conclusions based 122 nan understanding of the rules and a good mathematical concept, this shows that the structure of S2’s thinking on the problem is well organized, so the researcher does not provide defragmenting interventions for solving problem number 2. However, defragmenting interventions are given for problems number 1 and 3 based on the results of the review as follows.

Problem Number 1

In this problem, the specific indicator of the PISA problem that is applied is the ability to understand the relationship between the language of the problem, the symbolic and formal language needed to represent it mathematically. This problem specifically asks to understand the context of mathematical information and present it in diagrammatic form. S3's inability to answer problem number 1 correctly indicates that there was an error in understanding or solving the given problem. The defragmenting intervention was given to problem number 1 in the problem, through the following review results.

(1) Dik : -Memori stick memiliki kapasitas 1 GB (1.000 mb)

- musik	550 mb
- Foto	338 mb
- Ruang bebas	112 mb

- membeli stick baru 2 GB (2000 mb)

Dit : grafik yang menunjukkan status disk stick memori baru ?

Penyelesaian : ~~2000 - 550 = 1450, 2000 - 338 = 1662~~

~~2000 - 112 = 1888~~

$550 + 338 + 112 = 1.000$ $2.000 - 550 = 1450$

$\frac{1.000}{2.000} = 2$ $2.000 - 338 = 1662$

Jadi pilihan bagian C *

$2.000 - 112 = 1888$

Translation

Given :

The memory stick has a capacity of 1 Gb (1000 mb)

- Music = 550 Mb
- Photo = 338 Mb
- Free space = 112 Mb
- New memory stick = 2 Gb

Asked :

Graph showing the disk status of a new memory stick ?

Answer :

$$550 + 338 + 112 = \frac{1.000}{2.000} = 2$$

$$2.000 - 550 = 1450$$

$$2.000 - 338 = 1662$$

$$2.000 - 112 = 1888$$

So the choice is C

Figure 5. S2's written answer to problem number 1

From the written answers in [Figure 5](#), it can be seen that S3 writes down what is known and asked in the problem using language that is understood. By comparing the written answer sheets and conversations during the interviews conducted, the researcher concluded that S3 did not fully understand the intent of the problem. This was found when the researcher further observed the answer sheets that were made. At completion, some steps were built incorrectly resulting in wrong answers.

The following is an interview excerpt that describes the S3 thinking structure errors in solving problems.

Q : What do you know about the problem?

S3 : (reading the problem) For number 1, note that the memory size of this stick is 1 GB (1000 MB), then 500 MB of music space, 338 MB of photo space, and 112MB of free space, then 2000 MB of new memory.

Q : What does the problem ask ?

S3 : What the problem asks about is a diagram showing the memory status

P : (Points at the picture in the problem) Do you really understand the diagram in question?

S3 : Understood sis.

P : Ok now if you understand the problem being asked, how do you solve it?

S3 : The step I use is to add up all the music space, photo space, free space, then the results are shared with the new memory stick.

Q : Can you explain the purpose of doing that step?

S3 : (laughs with confused expression) uh... so what...? I'm just trying sis.

From the interview excerpt above, it was concluded that S3 did not fully understand the information and problems intended by the problem. It was seen that he was only able to re-read the text of the problem and did not reveal that there was free space capacity information that changed after the new memory was filled with music and photo components. This can be caused by a lack of literacy skills in reading problem tests. Furthermore, S3 was also unable to formulate a plan for completing it properly. S3 understands that the problem the problem

wants is how the memory capacity is presented in the form of a pie chart. However, S3 is confused about making the right mathematical calculation formulation to describe this memory capacity in a pie chart.

As a result, S3 takes a guessing step by adding up all the pieces of memory, then dividing by the new memory capacity. Furthermore, when asked to continue the completion steps from before, S3 writes out the steps that are not related to each other. S3 writes the reduced capacity of the new memory with each of the contents of the old memory section.

From the steps and calculations above, S3 continues by drawing conclusions from the answers. The following are excerpts from interviews that describe these thinking errors.

Q : Why do you conclude that the answer to the problem is C?

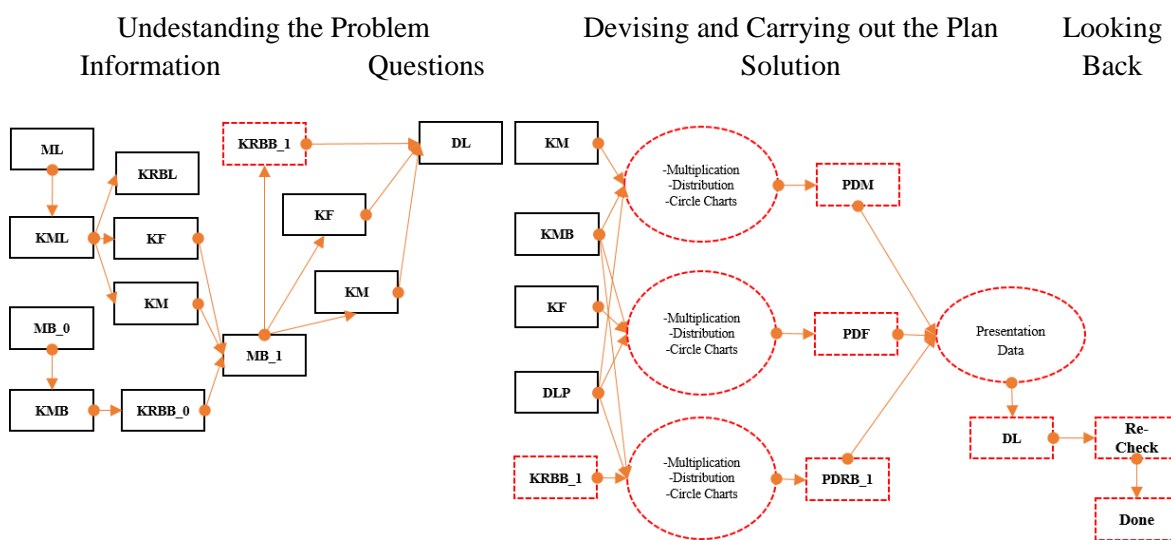
S3 : So..I just use my mind, sis, which image matches the size of the memory compartments.

Q : Are you sure about your answer?

S3 : Actually I'm not really sure sis, because they are almost the same size sis.

From the dialogue excerpt above, it is clear that the solutions made and the conclusions drawn are not interconnected, the conclusions drawn are only through alternative thoughts, by estimating the diagram according to the size of the memory content. However, what is estimated in the diagram presentation is wrong. . The estimates taken have a precision that is far from the actual size. In diagram C, the size of the music is > 50% capacity

The following is a description of the cognitive mapping of errors in the S3 thinking structure on problem number 1 presented in Figure 6 below.



Description

- | | | |
|------------------------------------|--|---|
| ML : Old memory description | KRBB_0 : Free space capacity new memory | DLP : Total circle degree portion |
| KML : Old memory capacity | MB_1 : The new memory description after the file is filled | Re-Check : Re-check answers |
| MB_0 : New free memory description | KRBB_1 : Free space capacity new memory once filled | □ : Components in Devising a plan |
| KMB : New memory capacity | DL : Pie chart | ○ : Components in Carrying out the plan |
| KM : Music file capacity | PDM : The proportion of the | ⊖ : Fragmented parts |
| KF : Photo file capacity | | |
| KRBL : Space capacity | | |

long memory free	degree of the music file
PDF : The proportion of the degree of the photo file	PDRB_1 : The proportion of degrees of free space new memory after filled with files

Figure 6 . S3 thinking structure error mapping on problem 1

From the results of the mapping of the thinking structure in [Figure 6](#), it shows that S3 failed to fully understand the problem information . S3 only focuses on KML components and fails to manage KF and KM information on ML to find KRBB_1 on MB . This case is in line with what was found ([Rofi'ah et al., 2019](#)) states that students focus on only one information question, so that they forget the information in the problem must also be answered (developed), this can happen because they do not record complete and thorough problem information data . In addition to problems in understanding information, apart from that there is a cognitive hole in the concept of presenting data so that S3 is not able to find the exact steps and processes for determining DL.

Based on the results of the S3 error diagnosis in solving problem number 1, the initial step taken by the researcher in this defragmenting process was to intervene to rearrange the S3 thinking structure to find KRBB_1 on the MB.

The following is an excerpt of the defragmenting interview given by the researcher to S2:

- Q : From 1 gb of old memory capacity, how much total memory is used? (Scaffolding-Restructuring)*
- S3 : ee.. 558 mb + 338 mb = 888 mb sis. (cognitive emergence)*
- P : OK, if the contents of the old memory are inserted into the new memory, how much will the new memory contain? (Scaffolding-Restructuring)*
- S3 : the same sis.*
- Q : What's the difference with the old memory? (Scaffolding-Restructuring)*
- S3 : Free memory sis*
- Q : How much free memory? (Scaffolding-Restructuring)*
- S3 : (Thinks for a moment while doing a calculation operation) er...one thousand one hundred and twelve. (cognitive emergence)*
- P : Well, now that you know, the contents of the components are in new memory, try to note down the parts that are known from the new memory after it is filled, so you don't forget it again (scaffolding-reviewing). Now from these components, how do you change them into pie chart (Scaffolding-Restructuring)*
- S3 : Can I use an estimate, sis?*
- Q : Yes, but with proper explanation.*
- S3 : Option A is wrong because the music space is bigger than the free space, C is also like that. So the answer is between option B and option D.*
- Q : So, are you confused? Have you ever studied material for presenting data in a pie chart?*
- S3 : Yes, sis, but I forgot the material.*

From the dialogue excerpt above, the researcher provides a Scaffolding-Restructuring intervention in the form of questions that lead S3 to develop and discover KRBB_1. When the researcher asked S3 to present KM, KF and KRBB_1 into DL, S3 reasoned the available options in the problem by looking for a match between the size of each component in MB_1 and the size of each in the pie chart available in the answer options. As a result, S3 experienced

confusion in determining the 2 choices between B and D because both of them met the criteria for the largest and smallest order set by S3 in matching the diagram. This happened because of the cognitive hole experienced by S3, especially in the material knowledge of pie chart data presentation.

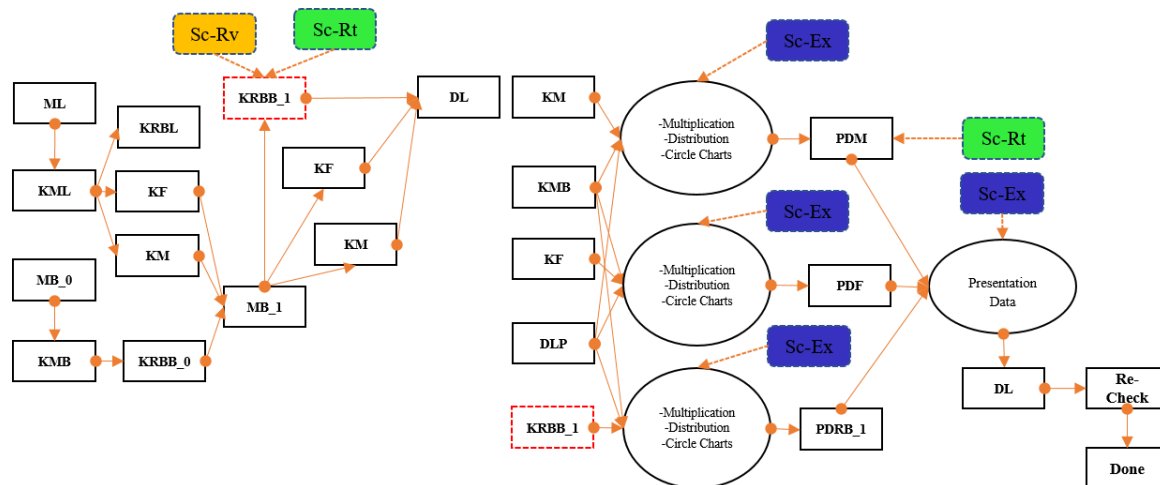
Therefore, the researcher decided to provide a scaffolding-explaining intervention in the form of an explanation of the concept and technique of presenting data in a pie chart starting from the introduction of DLP, the steps for calculating PDF and PDM, then how to present the proportion of degrees into DL to S3. good explanation given. This success can be seen when S3 starts trying to apply the explanation provided to the problem.

The following is an excerpt from the defragmenting interview given by the researcher to S2.

P : Try to determine how many degrees of proportion for music? (Scaffolding-Restructuring)
S3 : (Thinks for a moment while doing a calculation operation) like this sis.. $\frac{550}{2000} \times 360^\circ = 99^\circ$
P : Yes, that's right.. do it in the same way for the other components.
S3 : Oh yes sis.

From the dialogue excerpt above, the previous scaffolding-explaining and scaffolding-restructuring interventions succeeded in bringing S3 to find the PDF and PDM of MB_1 . The success of this intervention brought S3 to conclude the answer correctly that the appropriate DL indicated status MB_1 was DL option C. From the defragmenting intervention provided as described above, S3 managed to build the correct conclusion with the correct solution.

The overall provision of the s3 defragmenting intervention on problem number 1 is presented in Figure 7 below.



Description

- Sc-Ex : Scaffolding-Explaining
- Sc-Rt : Scaffolding-Restructuring
- Sc-Rv : Scaffolding-Reviewing

Figure 7 . Defragmenting S3 on issue 1

Problem Number 3

In this problem, the specific indicator of the PISA problem that is applied is the ability to calculate the start time for a trip given two different speeds, the total distance to travel and the end time. This problem specifically asks for an explanation of the proof of statements made in a mathematical context. On this problem S3 answered, but was not accompanied by the correct mathematical rules, this indicated that there was an error in understanding or solving the given problem. The defragmenting intervention was given to problem number 1 in the problem, through the following review results

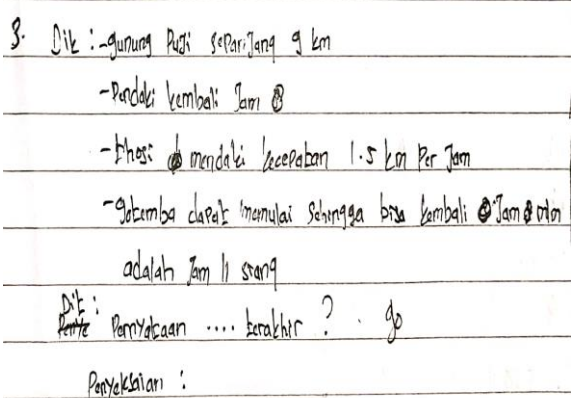
	<p>Translation</p> <p>Given :</p> <ul style="list-style-type: none"> -Mount Fuji is 9 km long -Climbers return at 8 o'clock -Thosi climbs at a speed of 1.5 km/ hour -Gotemba can start so can return at 8 pm is 11 pm. <p>Asked: Finalstatement?</p> <p>Answer :</p>
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Figure 8. S3's written answer to problem number 3

Based on the written answers in [Figure 8](#), it can be seen that S3 can write down what is known and asked in the problem using language that is understood. Even though it was incomplete, during the interview process, S3 was able to explain what information was contained in the problem text given. This shows that the S3 thinking structure is clear and structured in understanding the problem. However, the completion section is not filled in at all and there is no visible solution from the writing. From the results of the interviews conducted by the researchers, it was shown that S3 really had difficulty in compiling steps to solve problem number 3.

The following is an interview excerpt that describes the difficulty of thinking S3 in solving problems.

Q : What do you know about the problem?

S3 : Mount Praise 9 km, must return at 8 pm, speed 1.5 km/hour. Dropped 2 times before.

Q : What does Mount Praise 9 km mean?

S3 : The distance, sis.

Q : What speed did you mention earlier?

S3 : Speed up 1.5 km, speed down 2 x before

Q : Means the speed of descent how much?

S3 : 1.5 km/hour x 2 = 3 km/hour

Q : Is there any other information?

S3 : That's all sis.

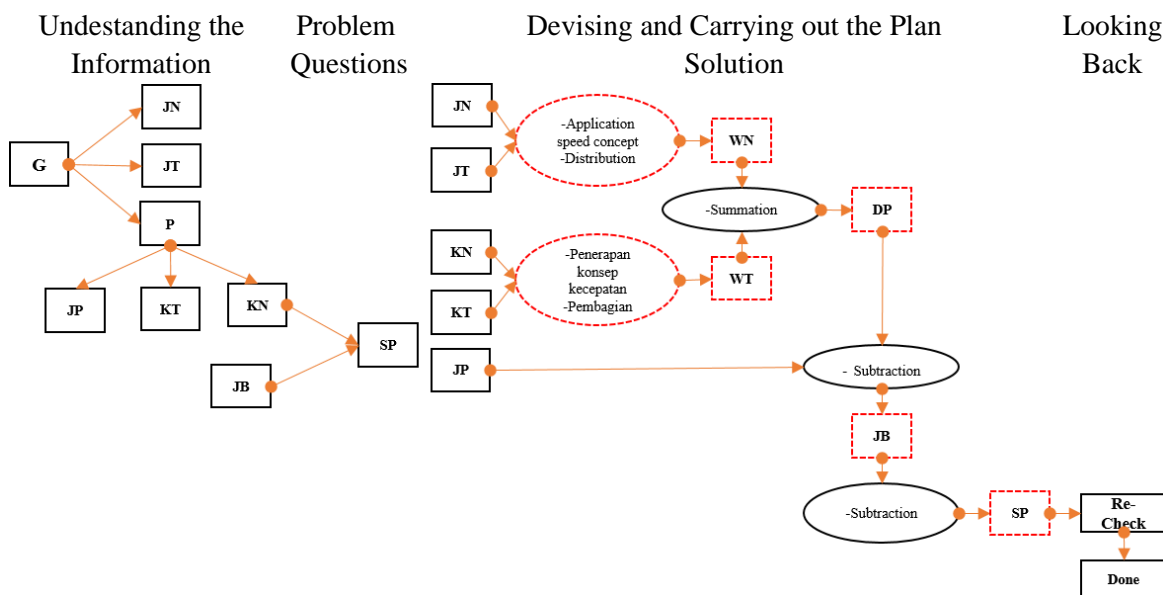
Q : What is the problem asking ?

S3 : Is it true that when you leave at 11 am, Gotemba can come back at 8 pm.

Q : If it's information and a problem that is facing a problem, why can't you continue solving it?
 S3 : I don't understand the formulas for time and speed, but wait a minute, let me try (do the calculation $9 \times 1.5 = 13.5$)
 Q : What do you mean you're working on?
 S3 : I'm just trying, sis, maybe it's wrong..
 Q : 13.5 what is the result?
 S3 : The duration of the trip is 9 km, sis.
 Q : After that, what else will you do?
 S3 : This is all I can do, I'm confused Sis, how to proceed.

From the interview excerpt above, it can be seen that S3 is able to understand the information and problems from the questions, but seems unable to formulate a solution plan correctly. S3 understands that the problem the question wants is to test the truth of the statement that if Gotemba starts climbing at 11 am, is it possible to come back at 8 pm. S3 was confused about making a resolution plan for the problem, so at this stage, S3 did not continue writing. When S3 is asked to retry making a solution, S3 tries to perform the operation of multiplying distance and speed with the aim of obtaining a travel duration of 9 km, in this case, S3 shows one of the correct plans but in the wrong way.

The following is a description of the cognitive mapping of errors in the S3 thinking structure in problem number 3 as shown in Figure 9 below.



Description

G : Description of the mountain
 JN : Distance up
 JT : Distance down
 KN : Speed up
 KT : Speed down
 Q : Climb
 JP : Time to go home Climbing

JB : Time to leave for the hike
 SP : Proof of statement
 WN : The time required during climb
 WT : The time required during down
 DP : The duration of the climbing trip

Re-Check: Re-check answers
 [Box] : Components in Devising a plan
 [Oval] : Components in Carrying out the plan
 [Dashed Box] : Fragmented parts

Figure 9 . S3 thinking structure error mapping on problem 3

From the results of the mapping of the thinking structure in Figure 9, it shows that S3 has been able to understand each component of what information is known and what is being asked from the questions. However, due to an incorrect understanding of the application of the speed concept, it caused S3 to fail in drawing up a plan to find WN, WT, DP and JB so that it was able to show SP.

Based on the results of S3's diagnosis in solving problem number 3, the initial step taken by the researcher in this defragmenting process was to intervene to sensitize the error in thinking about the solution that was made, besides that S3 requires some knowledge of applying the concept of speed.

The following is an excerpt of the defragmenting interview given by the researcher to S3:

- P* : Listen carefully, does multiplying the average speed by the distance yield the travel time, for example the speed is 2 km/hour, the distance is 4 km, $2 \times 4 = 8$ means the travel time is 8 hours (cognitive conflict)
- S3* : Eee.. not 2 hours sis.
- P* : That's right, you mean to find the formula for the travel time not by multiplying the distance traveled by the average speed but it should be? (scaffolding-restructuring).
- S3* : Divide the distance traveled by the average speed sis.
- P* : Now, pay attention to the solution you wanted to make earlier, does the multiplication of 9 km x 1.5 km/hour show the travel time? (Scaffolding-Review)
- S3* : Oh yes sis, it means this is wrong, it should be $\frac{9 \text{ km}}{1,5 \text{ km/jam}}$. (S3 cannot perform decimal division operation)

From the dialogue excerpt above, the researcher provides a cognitive conflict intervention in the form of giving an example that can refute the S3 understanding in applying the concept of speed. Through this intervention, S3 was able to realize his previous thinking mistakes. Furthermore, through the scaffolding-restructuring intervention in the form of guiding questions, S3 was able to find the speed concept formula in determining WN and WT through the examples given. After the intervention was successful, the researcher provided further intervention through scaffolding-review, by asking S3 to re-correct the solution previously made. However, when operating calculations $\frac{9 \text{ km}}{1,5 \text{ km/jam}}$, S3 cannot perform divisions that contain decimal elements. Therefore, the researcher decided to provide a scaffolding-explaining intervention in the form of technical explanations for division that contains decimal numbers to overcome the cognitive gaps experienced. Once it is known to fully understand what is described, subsequent defragmenting efforts lead S3 to find JN, JT and DP.

The following is an excerpt of the defragmenting interview given by the researcher to S3:

- Q* : (Sketch the mountain) Look at this picture, the distance from the foot of this mountain to the top is 9 km. The ascending speed is 1.5 km/hour. How long does it take to climb the mountain? (Scaffolding- Restructuring)
- S3* : 6 hours sis
- P* : Now, how long does it take to get down (Scaffolding-Restructuring).
- S3* : Oh I understand sis, 9 km is also the distance to go down sis, it means $\frac{9 \text{ km}}{3 \text{ km/jam}} = 3 \text{ jam}$ it takes time to get down.
- P* : Now, how long does it take during the climb? (Scaffolding Restructuring)
- S3* : $6 + 3 = 9$ hours sis
- Q* : What about the problem question? (Scaffolding Restructuring)

S3 : (Thinks for a moment to calculate the time duration from 11 am to 8 pm) Ooh, your statement is correct sis.

Q : Are you sure the statement is true? (Disequilibrium)

S3 : Sure sis, because 6 hours up and 3 hours down, it means the climbing time is 9 hours. From 11 a.m. – 8 p.m., the duration is 9 hours, it means the statement is correct sis.

From the dialogue excerpt above, the researcher provides an intervention in the form of scaffolding-restructuring in the form of questions that lead S3 to find the right solution. The first scaffolding-restructuring was successful in directing S3 to find WN. The second scaffolding-restructuring was successful in directing S3 to find WT. The third scaffolding-restructuring was successful in directing S3 to find DP. The fourth scaffolding-restructuring was successful in leading S3 to correctly conclude the SP. Furthermore, the researcher provided a disequilibrium intervention in the form of questions that indicated a doubtful attitude towards the answers given. The intervention provided does not create gaps in thinking about the answers given. This shows that, S3 has been in a state of balance in thinking (equilibrium) the balance between assimilation and accommodation obtained. From the results of several defragmenting interventions provided as described above, S3 managed to build the right conclusions with the right solutions.

The overall administration of the s3 defragmenting intervention on problem number 3 is presented in Figure 10 below.

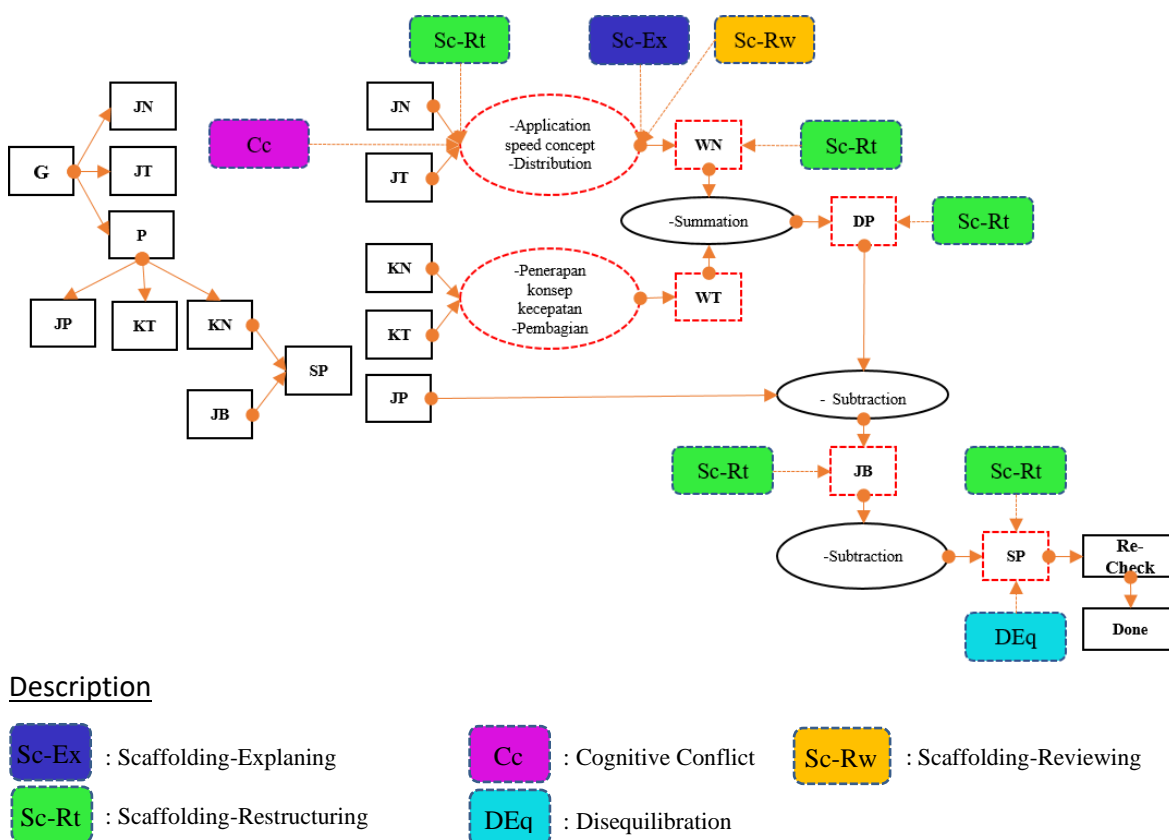


Figure 10 . Defragmenting S3 on issue 3

Discussion

This study revealed significant variation in thought structure errors and defragmentation strategies among the three participants. The findings showed that the only participants who were able to answer all the questions correctly were those who had high conceptual understanding. In line with research results. In line with the results of research (Kholid & Ahadiyah, 2022) that only students with high comprehension abilities are able to solve the problems given properly. This is because the structure of thinking in solving the problem is well organized (Subanji, 2015). Students with a high level of understanding are able to fulfill all aspects in a good category, starting from understanding the problem, planning to carry out the plan, re-examining the solution to the problem (Ani & Rahayu, 2018). These considerations show that students with a good understanding of concepts no longer need the intervention of defragmentation of thinking structures to solve the problems in the questions.

The correctness of the student's answer to the question cannot represent the absolute correctness of the student's understanding. As happens to students with medium ability categories. The occurrence of pseudo-thinking that builds understanding. According to (A. Wibawa, 2017) pseudo occurs because students do not reflect on the answers given, there is a tendency to only pursue the similarity of the problem with the potential for a wrong solution. In addition, the absolute factor that can trigger is a lack of understanding of mathematical concepts, this is mentioned by (Vinner, 1997) as pseudo-conceptual. In this process, the first step given when students find they are making mistakes is to try to make students aware of the errors in their thinking structure. These efforts in the defragmenting process can be carried out through cognitive conflict intervention. Giving cognitive conflict interventions in the form of giving similar examples to refute understanding (Subanji, 2016) is able to immediately make students realize that what they previously understood was incorrect. Cognitive conflict is successful because students are directed to think to reflect back on the answers given by comparing the simple examples given. Furthermore, providing scaffolding-restructuring and explanatory scaffolding interventions, respectively in the form of guiding questions and explanations, is able to direct students to solve the problems given correctly. These results are in line with research (A. Wibawa, 2017) which states that false thinking errors can be overcome through defragmenting intervention.

The literacy ability to read the question text causes errors to trigger errors at all stages of working on the question, starting from understanding the information, understanding the problem, preparing a solution plan and errors in carrying out the solution. This is also in line with findings (Rahman & Iyan Rosita Dewi, 2021) that errors in understanding information and problems known from the questions can be caused by students' limited reading abilities. comprehension skills. Apart from that, understanding prerequisite material is also the basic foundation of students' ability to solve problems. Students with a weak foundation of prerequisite material are unable to make solutions correctly even though the problem has been understood. This analysis is strengthened by research presentation (Rahman & Iyan Rosita Dewi, 2021) that errors in completion can be caused by a lack of knowledge of the concept of the material being studied. In these conditions efforts to rearrange understanding were carried out with scaffolding restructuring interventions in the form of questions or directions that guided participants to gain proper understanding (Buhaerah, 2022) succeeded in bringing up new cognition of the information and problems given by the problem, but due to the lack of knowledge about prerequisite material making it difficult follow-up interventions that are more

independent in nature, require the provision of scaffolding-explaining which refers to (Anghileri, 2006) and (Abadi et al., 2017) in the form of providing an explanation of understanding of a concept that is considered fatal.

Understanding the prerequisite material can also have a big impact on the steps in the completion process, this is because the prerequisite material will be the cognitive part needed in preparing a completion plan. This analysis is strengthened by the statement (Utami & Wutsqa, 2017) that students' ability to plan and carry out problem solving can be seen through understanding formulas or material concepts. In the defragmenting process, the first step is to be aware of errors in the thinking structure of the solution plan that has been made. The initial effort given was to provide examples that could invalidate the settlement arguments made, this intervention was categorized (Subanji, 2016) as cognitive conflict. The success of this process is marked by students becoming aware of the thinking errors they have made. In such conditions, scaffolding-restructuring intervention assistance in the form of structured questions can guide students' thinking to find the correct concept. Providing interventions that direct re-checking of completion can be done using scaffolding which is categorized (Anghileri, 2006) as scaffolding-review. Furthermore, in conditions of limited understanding of the material prerequisites, it complicates the structuring process, so that scaffolding-restructuring assistance is needed which provides questions that lead to a correct understanding of the concept. However, when providing this intervention is not enough, the final step that can be provided is to provide assistance by providing scaffolding-explaining in the form of assistance in providing direct explanations. Furthermore, asking questions that show doubt about the answers given as referred to by (Damayanti et al., 2020) as disequilibrium can test and check understanding after the intervention. According to (Subanji, 2016) success in the defragmenting process is marked by the participant's ability to balance their thinking condition, namely the balance between assimilation and accommodation that has been obtained, this condition is called the balance condition.

The research carried out has not paid attention to many other factors that can also influence students' thinking structures, including the type of student learning style and the character and personality of each student, so it is likely that further research can be carried out. pay attention to other factors as mentioned, apart from that the limited participants can result in a lack of abundant data to generalize research findings. So researchers also suggest that similar research can increase the number of participants so that it is hoped that many new facts or findings will be discovered from the research results.

Conclusion

Based on the results of the analysis, this research revealed significant differences in thinking structure errors between participants, with the discovery that limited literacy, a weak basis for creating material, the level of understanding of concepts, played a key role in the participants' ability to organize their thinking structures in solving problems. Defragmentation interventions, especially through the use of various types of scaffolding such as scaffolding-restructuring, scaffolding-explaining, scaffolding-review, and customized cognitive conflict interventions have been proven to help participants improve their thinking. However, this study recognizes the limitations of considering additional factors, and encourages further research with a larger number of participants and consideration of individual factors such as student learning styles. This conclusion provides an in-depth view of the potential of

defragmentation interventions in the context of students' understanding and solving problems in the learning process.

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References

- Abadi, A. P., Subanji, S., & Chandra, T. D. (2017). Diagnosis kesulitan siswa dalam menyelesaikan masalah geometri-PISA melalui pemetaan kognitif dan upaya mengatasinya dengan scaffolding. *MENDIDIK: Jurnal Kajian Pendidikan Dan Pengajaran*, 3(1), 1–18. <https://doi.org/10.30653/003.201731.71>
- Ackermann, B. F., Eden, C., & Cropper, S. (2004). *Getting started with cognitive mapping*. April 1992, 1–14. Retrieved from <https://banxia.com/pdf/de/GettingStartedWithCogMapping.pdf>
- Angraini, D., Kusmayadi, T. A., & Pramudya, I. (2018). The characteristics of failure among students who experienced pseudo thinking. *Journal of Physics: Conference Series*, 1008(1). <https://doi.org/10.1088/1742-6596/1008/1/012061>
- Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning. *Journal of Mathematics Teacher Education*, 9(1), 33–52. <https://doi.org/10.1007/S10857-006-9005-9>
- Ani, E. U., & Rahayu, P. (2018). Profil kemampuan pemecahan masalah matematika siswa berbentuk soal cerita materi bangun ruang. *Jurnal Riset Pendidikan Dan Inovasi Pembelajaran Matematika (JRPIPM)*, 1(1), 40. <https://doi.org/10.26740/jrpipm.v1n1.p40-49>
- Brender, J. (2006). *Framework for Meta-Assessment of Assessment Studies* Ethnographic research.
- Budhwar, P. S. (1996). Cognitive mapping as a tool to elicit managerial cognitions : Methodology analysed. *Journal of Conflict Resolution*, 21(4), 17–25. <https://doi.org/10.1177/0022002796040003001>
- Buhaerah, B. (2022). Scaffolding through cognitive mapping based on diagnosing student's difficulties in solving problem. *Journal for the Education of Gifted Young Scientists*, 10(8), 207–220. <https://doi.org/10.17478/jegys.1099807>
- Damayanti, P. A., Subanji, S., & Sukoriyanto, S. (2020). Defragmentasi struktur berpikir siswa impulsif dalam memecahkan masalah geometri. *Jurnal Pendidikan: Teori, Penelitian, Dan Pengembangan*, 5(3), 290. <https://doi.org/10.17977/jptpp.v5i3.13239>
- Djadir, Awi, S. (2018). Deskripsi kesalahan siswa dalam menyelesaikan soal matematika berbasis pisa pada konten change and relationship. *Issues in Mathematics Education*, 2(2), 112–118. Retrieved from <http://www.ojs.unm.ac.id/imed>
- Ena, Z. (2020). *Defragmenting struktur berpikir siswa dalam mengonstruksi konsep matematika pada materi aljabar* (Thesis) Retrieved from <http://repository.iainambon.ac.id/1210/1/BAB%20I%2C%20III%2C%20V.pdf>
- Estes, W. K. (2014). Handbook of learning and cognitive processes: Volume 5 human information processing. *Handbook of Learning and Cognitive Processes: Volume 5 Human Information Processing*, V, 1–337. <https://doi.org/10.4324/9781315770314>
- Fadhilah, L. F., & Rosyidi, A. H. (2020). Analisis kesalahan siswa dalam menyelesaikan soal pisa kategori formulate. *MATHEdunesa*, 9(1), 73–81. <https://doi.org/10.26740/mathedunesa.v9n1.p73-81>
- Fazzilah, E., Effendi, K. N. S., & Marlina, R. (2020). Analisis kesalahan siswa dalam menyelesaikan soal pisa konten uncertainty dan data. *Jurnal Cendekia : Jurnal Pendidikan Matematika*, 4(2), 1034–1043. <https://doi.org/10.31004/cendekia.v4i2.306>
- Haryanti, S. (2018). Pemecahan masalah matematika melalui metode defragmenting. *JKPM (Jurnal Kajian Pendidikan Matematika)*, 3(2), 199. <https://doi.org/10.30998/jkpm.v3i2.2768>
- Kholid, M. N., & Ahadiyati, A. (2022). Students' metacognition in solving non-routine problems. *Al-Jabar : Jurnal Pendidikan Matematika*, 13(1), 125–138. <https://doi.org/10.24042/ajpm.v13i1.11776>

- Kumalasari, F., Nusantara, T., & Sa'dijah, C. (2016). Defragmenting struktur berpikir siswa dalam menyelesaikan masalah pertidaksamaan eksponen. *Jurnal Pendidikan: Teori, Penelitian, Dan Pengembangan*, 1(2), 246–255. <http://dx.doi.org/10.17977/jp.v1i2.6129>
- Kurniati, D., Harimukti, R., & Jamil, N. A. (2016). Kemampuan berpikir tingkat tinggi siswa smp di kabupaten jember dalam menyelesaikan soal berstandar PISA. *Jurnal Penelitian Dan Evaluasi Pendidikan*, 20(2), 142–155. <https://doi.org/10.21831/pep.v20i2.8058>
- Labuschagne, A. (2015). Qualitative Research - Airy Fairy or Fundamental? *The Qualitative Report*, 8(1), 100–103. <https://doi.org/10.46743/2160-3715/2003.1901>
- Lundgrén-Laine, H., & Salanterä, S. (2010). Think-aloud technique and protocol analysis in clinical decision-making research. *Qualitative Health Research*, 20(4), 565–575. <https://doi.org/10.1177/1049732309354278>
- Lutfianto, M., Zulkardi, & Hartono, Y. (2013). Unfinished student answer in Pisa mathematics contextual problem. *Journal on Mathematics Education*, 4(2), 188–193. <https://doi.org/10.22342/jme.4.2.552.188-193>
- Mardhiah, M. Z., Mailizar, M., & Elizar, E. (2021). Students' mathematics problem-solving skills in PISA problems. *AIP Conference Proceedings*, 2331(April). <https://doi.org/10.1063/5.0041685>
- Matthew B. Miles, A. M. H. (1984). Qualitative Data Analysis. In *International Legal Materials* (Vol. 33, Issue 6). SAGE Publication. <https://doi.org/10.1017/s0020782900036494>
- OECD. (2011). *Lessons from PISA for the United States Strong Performers and Successful Reformers*. <https://doi.org/http://dx.doi.org/10.1787/9789264096660-en>
- Polya, G. (1978). How to solve it: a new aspect of mathematical method second edition. In *The Mathematical Gazette* (Vol. 30, p. 181). Retrieved from <http://www.jstor.org/stable/3609122?origin=crossref>
- Rahman, R. F., & Iyan Rosita Dewi, N. (2021). Analisis Kesalahan Siswa Dalam Menyelesaikan Soal Pemecahan Masalah Matematika Berdasarkan Teori Newman. *Hipotenusa Journal of Research Mathematics Education (HJRME)*, 4(2), 102–113. <https://doi.org/10.36269/hjrme.v4i2.499>
- Roebyanto, G., & Harmini, S. (2016). *Pemecahan Masalah Matematika Untuk PGSD* (1st ed.). Universitas Negeri Malang.
- Rofi'ah, N., Ansori, H., & Mawaddah, S. (2019). Analisis Kesalahan Siswa Dalam Menyelesaikan Soal Cerita Matematika Berdasarkan Langkah Penyelesaian Polya. *EDU-MAT: Jurnal Pendidikan Matematika*, 7(2), 120. <https://doi.org/10.20527/edumat.v7i2.7379>
- Rolka, K., & Liljedahl, P. (2007). The role of cognitive conflict in belief changes. *31st Conference of the International Group for the Psychology of Mathematics Education*, 4, 121–128.
- Sa'adah, A., Misri, M. A., & Darwan. (2019). Analisis kesalahan siswa dalam menyelesaikan soal matematika HOTS bertipe PISA. *Journal For Islamic Social Sciences*, 3(1), 53–64. <https://doi.org/10.24235/holistik.v3i1.5566>
- Sasongko, D. F., Subanji, & Sulandra, I. M. (2018). Deskripsi kesalahan struktur berpikir siswa smp dalam menyelesaikan masalah geometri serta defragmentingnya: Suatu Studi Kasus. *Jurnal Kajian Pembelajaran Matematika*, 2(1), 10. Retrieved from <http://journal2.um.ac.id/index.php/jkpm>
- Siau, K., & Tan, X. (2017). Use of cognitive mapping techniques in information systems development. *Journal of Computer Information Systems*, June 2008. <https://doi.org/10.1080/08874417.2008.11646034>
- Subanji. (2015). *Teori Kesalahan Kontruksi Konsep dan Pemecahan Masalah Matematika*. UM Press.
- Subanji. (2016). *Teori Defragmentasi Struktur Berpikir* (1st ed., Issue February). UM Press.
- Tasni, N., & Susanti, E. (2017). Building students' mathematical connections in verbal problem solving. *Beta Jurnal Tadris Matematika*, 10(1), 103. <http://dx.doi.org/10.20414/betajtm.v10i1.108>
- Trevor R. Anderson, K. J. S. (2008). Bridging the educational research-teaching practice gap. *The International Union of Biochemistry and Molecular Biology*, 38, 309–315.
- Utami, R. W., & Wutsqa, D. U. (2017). Analisis kemampuan pemecahan masalah matematika dan self-efficacy siswa SMP negeri di Kabupaten Ciamis. *Jurnal Riset Pendidikan Matematika*, 4(2), 166. <https://doi.org/10.21831/jrpm.v4i2.14897>
- Vinner, S. (1997). The pseudo-conceptual and the pseudo-analytical thought processes in mathematics learning. *Educational Studies in Mathematics*, 97–129. <https://doi.org/10.1023/A:1002998529016>
- Wahab, A. (2022). Diagnosa kesulitan siswa dalam menyelesaikan soal spltv melalui pemetaan kognitif berbasis polya dan upaya mengatasi dengan scaffolding. *RAINSTEK: Jurnal Terapan Sains & Teknologi*, 4(1), 1–14. <https://doi.org/10.21067/jtst.v4i1.6377>

- Wibawa, A. (2017). Pseudo siswa dalam memecahkan masalah limit fungsi. *Nasional Exchange of Experiences Teacher Quality Improvement Program*.
- Wibawa, K. A. (2016). *Karakteristik berpikir pseudo dalam pembelajaran matematika*. March, 0–16. <https://doi.org/10.13140/RG.2.1.3907.3528>
- Zubaeda, E. (2020). *Defragmenting struktur berpikir siswa dalam mengonstruksi konsep matematika pada materi aljabar*.(Thesis) Retrieved from <http://repository.iainambon.ac.id/1210/1/BAB%20I%2C%20III%2C%20V.pdf>