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Research articles

Exploring students' imaginative process: Analysis, evaluation, and creation in mathematical problem-solving

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Abstrak Peran imajinasi sebagai sarana belajar matematika belum didefinisikan dengan baik, tidak seperti pada bidang lain contohnya seni dan sastra. Tujuan penelitian ini adalah menelusuri proses imajinasi siswa selama melakukan pemecahan masalah matematika. Penelitian ini melibatkan tiga siswa kelas 7. Pemilihan peserta penelitian dilakukan dengan menggunakan *purposive sampling*, berdasarkan nilai tertinggi hasil tes pemecahan masalah kreatif matematika. Jawaban siswa dan hasil wawancara dianalisis secara kualitatif dengan merujuk pada tiga tahapan proses kreatif yang melibatkan imajinasi: analisis, evaluasi dan kreasi. Hasil penelitian menunjukkan, pada tahap *analisis*, imajinasi yang terbentuk ditandai dengan kemampuan siswa menetapkan masalah secara umum *(common visual)*. Sebagai langkah awal untuk menyelesaikan masalah, siswa melakukan koreksi dengan cara memikirkan kembali pengetahuan matematika yang dibutuhkan. Proses imajinasi pada tahap *evaluasi* ditunjukkan oleh kemampuan siswa dalam menyimpulkan jawaban akhir dengan cara membangun visual dari pengalaman sebelumnya sebagai artefak yang diambil bersama serta mengumpulkan pengetahuan yang diperlukan. Pada tahap *kreasi*, kemampuan siswa melakukan proses berpikir secara siklis dalam memikirkan ide baru untuk menyelesaikan masalah yang dihadapi menunjukkan proses imajinasi pada tahap ini. Proses ini berlangsung berulang kali, sampai siswa mengambil keputusan bahwa tidak memiliki ide yang lain untuk menyelesaikan masalah.

Kata kunci *Pemecahan masalah, Imajinasi, Analisis, Evaluasi, Kreasi*

Abstract The role of imagination as a means of learning mathematics, unlike in other fields such as art and literature, is not well defined. The present study aims to examine the process of students' imagination in solving mathematics problems. It involved three grade 8 students which were purposively selected based on their scores in a given test. Students' answers to the test and the results of interviews were examined qualitatively referring to the three stages of creative problem-solving that involve imagination: analysis, evaluation and creation. The results show that, in the *analysis* phase, imagination was found in the students' ability to define problems in general (common visual). As the first step in solving a problem, they analysed mathematical knowledge needed to solve the problem. In the *evaluation* phase, imagination was formed as students completed the final answer by creating visual representations from previous experiences as artifacts taken together and gathering necessary knowledge. In the last phase, *creation*, imagination was identified when students engaged in a cyclical thought process to find new ideas in solving the problem. This process repeated until the students decided there was no other ideas or ways to solve the problem.

Keywords *Problem-solving, Imagination, Analysis, Evaluation, Creation*

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Introduction

Imagination and mathematics are frequently combined in research and scholarship, but their relationship is complex, constantly unexplained, and challenging to comprehend (Egan, 2013; Gadanidis, 2006; Arana, 2016; Lakoff & Núñez, 2000). Similar combinations have been used in mathematics education, where creativity is a crucial and valuable discussion topic (Davis & Simmt, 2006; Kasner & Newman, 2001; Mazur, 2004). Gadanidis (2006) explains that it is difficult to search the mathematics curriculum regarding the importance of imagination, "I can't find the word 'imagination' in this curriculum, and it's very troubling." Children have extraordinary imaginations, and they are able to use their imaginations to explore, question, seek interesting connections in learning mathematics. It remains difficult to establish a connection between the two, such as explaining the role of imagination in mathematics education, whether as a pedagogical tool or as a content of mathematics itself (Hilbert & Cohn-Vossen, 2021).

Research by Mazur (2004) compels us to reconsider whether teaching and learning mathematics can be used to explore imagination. In particular, we can question whether the process and steps of reapplying the imaginative work of mathematician could be used by high school students to solve mathematical problems in the classroom. Unlike other fields, such as art and literature, imagination as a method for learning mathematics has rarely been studied (Egan, 2013). The purpose of this study was to examine students' imaginative processes while solving mathematical problems. The research question sought to be answered was how does students' imagination function when solving mathematical problems?

Defining imagination in problem-solving

It is necessary to possess many skills in various aspects of life, such as practical problem-solving abilities. Problem-solving abilities are important to all mathematical education programs (Jitendra et al., 2007; Leikin, 2014; Ginsburg et al., 2009; Kwon et al., 2021; Polya, 2004). According to NCTM (2000), mathematical problem-solving is concerned with learning in general and, more particularly, the process of developing mathematical imaginative skills in their entirety. Not only mathematicians, but also students and teachers must use their imaginations to solve mathematical problems in the classroom. It is crucial to emphasize the necessity of creativity in solving math problems. Mathematical failure begins with the application of memorized formulas to problem-solving situations. In light of this, problem-solving should focus not just on comprehending the problem at hand, but also on devising a variety of solutions. For instance, students should participate in the process of identifying the core of the problem (e.g., analyzing the problem, correcting a given problem solution, evaluating by changing erroneous data in identifying the problem and constructing an entirely new problem solution). In other words, creativity is essential for acquiring a deeper comprehension of a math topic. Moreover, creativity encourages pupils to modify the problem-solving procedure or to develop a completely new answer. This activity is seen essential for the formation of future mathematicians.

Prior studies have demonstrated that problem-solving and imagination are inherently linked and mutually beneficial (Sarwanto et al., 2020; Rodionov, 2013; Hegarty & Kozhevnikov, 1999). The imaginative thinking process is a student-centered approach that helps students enhance their mathematical thinking skills (McFarland et al., 2017), build spatial thinking skills (Hegarty & Kozhevnikov, 1999) and increase their academic performance. It is essential to stress that innovative thinking is a vital component of educational programs that combine mathematics and science, in particular STEM programs, which are an integral aspect of education in many nations.

Generally, imagination and visualization are considered the same. However, Mazur (2004: 138) explains that "providing a natural home for that something in the mind's mind", that resists familiar "types of mental image-making activities". He further emphasizes the difference between the two by mentioning "imagine a . . . object and visualizing it can [our emphasis] be different." He explains that visualization is the process of bringing an image to "the mind's screen" using the "mental equipment" already present. Imagination, on the other hand, has no pre-existing mental apparatus or images, and its outcomes are unknown. The images identified by Mazur can be single objects (such as triangles), combinations of objects (such as multiple polygons, symbols, etc.), or experiences (e.g., watching a teacher-led demonstration).

The differences between asking students to engage in visualization and imagination based on whether the student is familiar with the image or object. If an image or object is unknown, students engage in imagination in which a new image or object is constructed in the mind. Instructing students to "imagine" a square, for instance, should result in a mental image of a regular four-sided polygon with equal sides and 90° internal angles. However, it may be more difficult to ask students to "imagine" a rectangle that could also be a square. Students may not have a mental image to facilitate comprehension. If students can visualize a rectangle also a square, they do not meet Mazur's definition of "imagination." They instead "visualize" it (Wellner, 2022).

This study conceptualizes imagination as a process that occurs in the cognitive space during problem-solving (Baars, 1997). When a student solves a problem and can understand the problem given by trying to imagine, it means that the student has entered the visualization process. When the student connects, differentiates, composes, and produces something that has never been imagined, it means that the student has entered a higher process, namely imagination. Visualization is part of the imagination process (Rodionov, 2013). Furthermore, Rodionov explains that visualization is imagining a result in the form of a general concept that has been found before. At the same time, imagination is creative thinking, namely the ability to think of something that has never been thought of before. In this cognitive space, visual stimuli are employed to activate pre-existing mental devices.

Imaginative process in creative problem-solving

Creative activity arises from a more comprehensive brain system and is linked to a variety of cognitive processes (Rodionov, 2013; Caselli, 2009). Many researchers believe that generating new ideas imaginatively is the defining characteristic of creativity (Arden et al., 2010). Without actualization, imagination is merely wishful thinking (Rahayuningsih, 2021). Moreover, Bértolo (2005) indicates that the function of imagination is to transform information from one modality to another, as blind individuals are able to describe objects based on their perceptions from birth. Blind individuals are able to perceive through the other five senses, including smell, motor sensations, and sound waves such as music (Mellet et al., 1999; Carrasco & Ridout, 1993). Imagination is a form of mental activity based on brain systems that re-integrate existing information in memory and new information received through diverse sensory channels (Radovanovic et al., 2002; Northoff et al., 2006). This statement implies a connection between perception and imagination. Perception is formed through the ability to analyze (a person's ability to distinguish/select), then evaluate (make decisions), and finally create (produce products that match the description), also known as the creative process (McFarland et al., 2017). According to Rodionov (2013), perception is formed through knowledge with the process of seeing, hearing, touching, feeling, and receiving something, followed by the process of selecting, organizing, and interpreting information into a meaningful picture.

The process of imagination is difficult to define, so we only observed expressions and gestures of students who seem to think and try to visualize something that is in their minds. When students are able to realize visualization images that exist in fantasy by producing a real output, such as generating unusual problem-solving ideas, it means that students are able to carry out good imaginative processes, even though anxiety and despair will appear. In this study, the imaginative process was identified when students showed three phases in solving problems (Rahayuningsih et al., 2021): analysis, evaluation, and creation. *Analysis* is the ability to disassemble a whole into parts and determine their relationships. Analysis entails a) distinguishing between irrelevant and relevant or important and unimportant parts of a given material; b) organizing or determining how an element fits and can function within a structure; and c) attributing or determining the core or underpinning of a given material. Evaluating is the process of making decisions based on standardized criteria, such as checking and criticism. The evaluation includes the following actions: a) checking or tracking the inconsistencies of a process or result; b) critiquing the inconsistency between the results and some external criteria or decisions according to the given problem procedure. Creating is the process of combining elements to form a coherent whole or generate an original product. Creating involves a) generating hypotheses based on the provided criteria; b) planning to complete a given task; and c) producing a product according to the given description. Table 1 presents the description of the imaginative process in this study.

Table 1. Indicators of the imaginative process

Level	*Description*
Analysis	• Students are able to analyze data to determine the value of Un if they know b from a sequence of numbers. • Students are able to analyze data to determine the *n*th term of a number sequence and Sn.
Evaluation	• Students are able to evaluate information to draw conclusions and identify reasons to support those conclusions. • Students are able to investigate/parse material to draw conclusions and find the arguments to support them.
Creation	• Students are able to create numerical sequences if they know Sn. • Students are able to create the formula of Un.

Methods

This study followed a qualitative approach which aims to collect information about the status of an existing symptom and describe systematically the facts and characteristics of the object or subject being studied appropriately and put it into words (Creswell, 2017). This article describes how students use their imagination to solve mathematical problems. The research data were collected through direct observation of the problem-solving procedure, a test of the students' imaginative abilities in comprehending problems, planning solutions, solving problems, and conducting evaluations. This description is also supported by the outcomes of interviews conducted after the test and student problem-solving assignments.

Participants and procedures

The research participants were first-semester eighth graders. A purposive sampling was used to select the participants, i.e., sampling was conducted with specific considerations or goals in mind (Creswell, 2017). The participants were determined by (1) selecting the grade level of the students (eighth grade in South Sulawesi) and (2) selecting a class to be administerred a imagination test based on the mathematical abilities of the students, i.e. selecting students with high mathematical abilities in the school. The subsequent steps were (3) administering a test to determine how students used their imaginations to solve mathematical problems; and (4) selecting the students who were the focus of the research by analyzing the results of the test. After analyzing the results of the test, three participants were selected based on their test scores, communication skills, and ability to express their thoughts while solving problems. According to Rahayuningsih (2023), students' creative thinking processes were evaluated from the participants' behavior, reflecting their mental activity in solving mathematical problems. Students' behavior was explored based on their written work and through in-depth interviews, so that good communication skills make it easier for the researchers to uncover students' thought processes.

Instruments

This study included primary and supporting instruments. The researchers were the primary instrument. In this study, we were the planner, implementer, data collector, data analyst, data interpreter, and, finally, the reporter of research findings. The supporting instrument was an imaginative test. The test was used to assess the participants’ imagination during mathematical problem-solving. It was constructed by taking into account all indicators of imagination, including analysis, evaluation, and creation. The test consisted of six questions, each two test examined the students’ ability to analyze, evaluate, and to create, respectively. In this study, the imagination test took the form of an essay test based on the applicable curriculum and the teacher-taught material. The following is examples of tasks.

Task 1 (Analysis)

Mr. Hadid is a manager in an insurance company. Last year he received a salary of IDR 15,000,000 per month. Because of his achievements he received a salary increase of IDR 750,000 so that this year he received a salary of IDR. 15.750.000,- per month. Next year his salary will increase again to Rp. 16.500.000,- per month. And so on he gets a salary increase of IDR 750,000 every year. If Mr. Hadid turns 40 this year, how much monthly salary will Mr. Hadid get when he is 54 years old?

Task 2 (Evaluation)

Andre is contracted to work for a company for 7 days. Before starting work, he was asked to choose between being given a salary of IDR 75,000 per day for a week, or being given a salary of IDR 10,000 on the first day and doubling every day for a week. Which is the best choice for Andre to choose so that he gets the maximum salary? Explain your answer!

Task 3 (Creation)

One hundred marbles will be placed in 10 different cups. Each cup contains a different number of marbles. So that the number of marbles in each cup forms an arithmetic series. What is the greatest number of marbles that can be placed in one of the cups?

Interview protocol

An interview guideline was used to facilitate the process of gathering in-depth information about students' imaginative process. This study employed unstructured interviews, which means that data were gathered using interview guidelines that were neither systematically nor totally structured. Interviews were performed using everyday language to encourage more responses from the participants. Open questions were also used to dig up in-depth information regarding students' imagination processes, for example what do you think, why do you think that, try to explain the reasons, or try to tell what you feel.

Data analysis

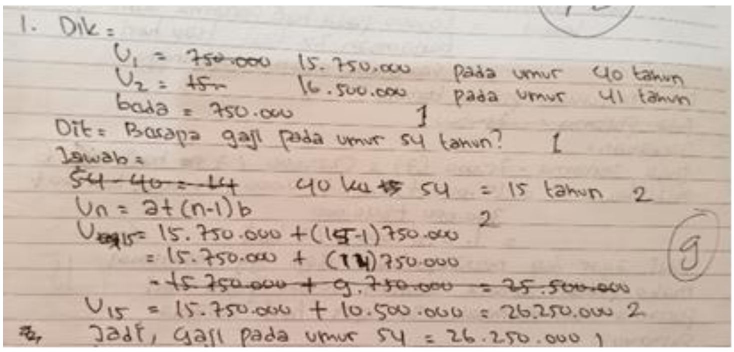
The results of the transcript and physical behavior exhibited by participants were analyzed with the following steps (Rahayuningsih, 2020):

- Examining all available data from various sources: interview transcripts and problem solving results sheets used to measure students' imaginative thinking skills.
- Carrying out data reduction by making abstractions. Abstraction is an attempt to make a summary of the core, process, and statements that need to be maintained to remain in it.
- Arranging data in segments which were further categorized by making coding.
- Checking the validity of data by means of time triangulation; Analysis of interesting cases, i.e. behavioral analysis shown by research participants who are unplanned and unrelated to the research objectives.
- Interpreting data and drawing conclusions.

Results and Discussion

Imaginative process in the analysis

Figure 1 shows the imagination ability of the first subject (S1) when answering question 1 (the *Analysis* phase).



Translation:
Given: U1 = 15.750.000 on 40 years old.
U2 = 16.500.000 on 41 years old.
difference = 750.000
Asked: Monthly pay when he is 54 years old?

Answer:
40 to 54 = 15 years
Un = a + (n-1) b
Un = 15.750.000 + (15-1) 750.000
Un = 15.750.000 + (14) 750.000
U15 = 15.750.000 + 10.500.000 = 26.250.000
So, when he is 54 years old, his monthly pay is 26.250.000

Figure 1. S1's answer to question 1 (Analysis phase)

To confirm the participant's answers, an interview was conducted. The results of the interview are presented in Table 2.

Table 2. The interview with S1

Code	p/j	Interview content
S1an11-001	P	*What can you understand from Question number 1?*
S1an26-010	J	*Pak Hadid is a manager, last year he earned Rp. 15.000.000/ month, because he got a rise Rp. 750.000, this year his salary increased into 15.750.000, -/ month. The year after, he got another rise Rp.750.000. into 16.500.000/ month. He got a rise in salary every year Rp. 750.000. the difference (n) is Rp. 750.000,-*
S1an26-030	P	*OK, very good, tell me more about it*
S1an30-002	J	*what is being asked is, Pak Hadid, is 40 years old, so what will his monthly pay be when he is 54 years old*
S1an30-004	P	*Very good. How do you solve the problem?*
S1an34-003	J	*First, I will determine the n from Pak Hadid's age, which is 54 years old. After that, I will use the arithmetic series formula to find out Pak Hadid's salary, because what was asked was the salary when Pak Hadid is 54 years old. So, I started counting n from the age of 40.*
Code	p/j	*Description*
S1an34-040	P	*Well, that makes sense, why is it from 40 to 54?*
S1an35-004	J	*Because the question mentions that Pak Hadid received Rp15.750.000. when he was 40 years old.* *The question asks his salary when he is 54 years old, then we need to find the gap between 40 and 54, the answer is 15 years old. I think I can use the formula* $U_n = a + b(n-1)$
S1an36-005	P	*Excellent, what is* U_n*?*
S1an36-006	J	U_n *is the nth term*
S1an36-007	P	*What is a?*
S1an36-008	J	*a is the first term*
S1an36-009	P	*What is n?*
S1an36-010	J	*the number of terms*
S1an36-011	P	*b?*

Code	p/j	Interview content
S1an36-012	J	*b is the difference*
S1an36-013	P	*How do you use this formula? (Pointing at the formula on the paper)*
S1an37-009	J	*Replace a with Rp. 15.750.000 and b with Rp.750.000, n is 15.*
S1an37-010	P	*Are you sure that your answer is correct?*
S1an39-010	J	*because a is Rp. 15.750.000 and n is 15, minus 1, multiplied by the difference which is Rp.750.000. The multiplication results in Rp. 26.250.000.*
S1an44-011	P	*Is there any way to solve the problem?*
S1an49-011	J	*There is, instead of using the formula, we can add the numbers*

Notes. S1 (subject 1), an (analysis), P (questions), J (answers)

The student's answer was responded by the interviewer with friendly remarks, such as "very good", "that's very intriguing" or "excellent." By reacting appropriately to the participant's answer, the interviewer provided a safe and open space for her to express herself, because her opinions and participation were valued (Lave & Wenger, 1991; Yackel & Cobb, 1996). As previous researchers have explained, this method keeps the classroom safe and allows students to express themselves freely. As a result, the classroom creates a more conducive environment for innovative and creative work, as students are more free to explore and make observations. This method is frequently taken for granted. This study also highlights the significance of listening to and valuing student responses as research participants in order to support and comprehend their learning process (Davis & Simmt, 2006; Yackel & Cobb, 1996).

In this study, we consistently asked students to describe their opinions, asked students to explain their thoughts, and provided examples to help them illustrate their ideas. If they were unable to provide clarification, we helped them comprehend the questions. Meaning is a continual negotiation that takes place reflexively through sharing.

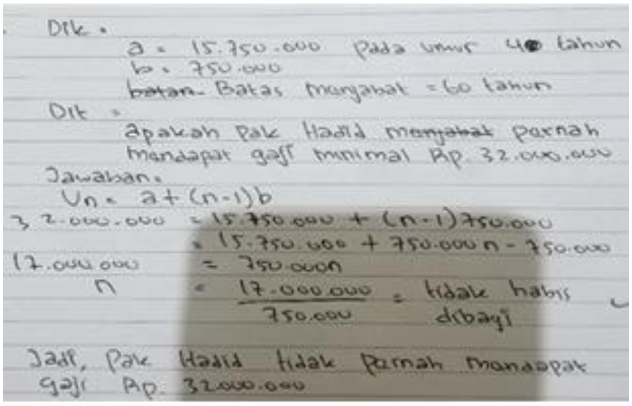
The interview excerpts above revealed that S1 was able to identify the known and asked components of the question provided. Also, S1 was able to deduce that the difference in salary earned per year was Rp. 750,000 and asked about Hadid's salary when he was 54 years old. When students are able to understand the questions offered, they are already in the analysis phase of the imaginative thinking process. Someone who is using the imaginative thinking process is able to come up with ideas that do not seem to make sense at first as solutions to problems.

S1 found a solution to the problem by comprehending the known and requested information, namely the *n*th term of Pak Hadid's age when he was 54 years old. We assumed that the student was "visualizing" the solution because the procedure took a considerable amount of time. After a period of silence, the student appeared to be fumbling with her answer sheets, occasionally rewriting and erasing her answers. It took over ten minutes for her to continue to start calculating Mr. Hadid's income. By calculating the difference between 40 and 54, which represented Mr. Hadid's ages, the student was able to predict the *n*th term when Mr. Hadid is 54 years old. S1 concluded that the problem could be solved using the arithmetic sequence formula. The *n*th term formula was derived through the problem-analysis process. The student initially did not anticipate using the formula, but after visualizing the problem, she was able to engage her mental devices. Visuals may consist of a description of events that students have had in the past (McFarland et al., 2017)

S1 was able to recall and comprehend the symbols contained in the arithmetic sequence formula, namely Un, a, and b, in order to solve the problem. The acquired data was then analyzed and substituted into an arithmetic sequence formula. In addition, S1 verified the accuracy of the response by recalculating Mr. Hadid monthly pay using the arithmetic sequence formula. In addition, S1 offered an alternative way to solve the problem, namely by manually adding up the numbers to determine Mr. Hadid's salary. Based on the results of observations on the student's imaginative processes at the analysis stage, it appears that imagination was formed when the student defined a general problem (common visual) as the first step to solving the problem, then correcting it by rethinking the required mathematical knowledge.

Imaginative process in the evaluation

Figure 2 shows the imaginative process of the second subject (S2) when answering question number 1 (the *Evaluation* phase).


Dik :
a = 15.750.000 pada umur 40 tahun
b = 750.000
~~batan~~ Batas menjabat = 60 tahun
Dit :
apakah Pak Hadid menjabat pernah mendapat gaji minimal Rp. 32.000.000
Jawaban:
Un = a + (n-1)b
32.000.000 = 15.750.000 + (n-1)750.000
= 15.750.000 + 750.000n - 750.000
17.000.000 = 750.000n
n = 17.000.000 / 750.000 = tidak habis dibagi
Jadi, Pak Hadid tidak pernah mendapat gaji Rp. 32.000.000

Translation:
Given: a = 15.750.000 on 40 years old
b = 750.000
term of office = 60 years old
Asked: Has Pak Hadid received minimum wage of Rp. 32.000.000, -?

Answer:
Un = a +(n-1) b
32.000.000 = 15.750.000 + (n-1) 750.000
= 15.750.000 + 750.000n – 750.000
17.000.000 = 750.000n
n = 17.000.000/750.000 = division is not possible
So, Pak Hadid never received the minimum wage of Rp. 32.000.000, -.

Figure 2. S2's answer to question 2 (Evaluation)

To confirm the participant's answer, an interview was conducted. The results of the interview are presented in Table 3.

Table 3. The interview with S2

Code	p/j	Interview content
S2ev22-001	P	*What can you learn from the question?*
S2ev30-001	J	*I learned that his salary last year was Rp 15,000,000 per month. Due to his accomplishments at work, he received a pay rise of Rp 750,000. This year, his monthly income is Rp 15.75 million. The compensation will increase by (current salary) Rp. 750,000, so he could earn Rp. 16,500,000/month. He receives a salary increase of Rp. 750,000 every year. If he is required to retire at age 60, he will no longer get a salary after reaching that age. Has Pak Hadid ever earned a minimum of Rp. 32,000,000/month? If so, when did he acquire it?*
S1ev32-002	P	*Wow, that's amazing, how did you solve the problem? What was the first step?*
S1ev32-002	J	*I determined a pattern, using the difference, which is 750.000.*
S1ev32-002	P	*Pattern? Good thought. After that?*
S1ev32-002	J	*This is an arithmetic sequence, so I used the formula Un = a + (n-1) b. I assumed 32.000.000 = a+ (n-1) b. If I can't get the exact 32.000.000, then it's impossible.*
S1ev32-002	P	*Try again.*
S1ev44-004	J	*(scratching) n equals 22.66666. He is 40 years old, plus 22, the result is more than 60, then it's impossible (for him to earn 32.000.000).*
S1ev32-003	P	*At what age can he earn 32.000.000 per month?*
S1ev32-004	J	*40 plus 22 equals 62 years old*
S1ev45-006	P	*What made the problem difficult?*
S1ev45-012	J	*To determine the age*

Notes. S1 (subject 1), ev (evaluation), P (questions), J (answers)

The interview excerpts demonstrate that S2 engaged in his "imagination" to comprehend the presented problem, as evidenced by his decision to halt and consider potential solutions (the process took quite a while). However, he was able to determine that Mr. Hadid's starting wage was Rp Rp. 15,750,000, that the difference in Pak Hadid's annual income was Rp. 750,000, and that the insurance company's retirement age was 60 years. S2 was able to identify the question being addressed, namely whether Mr. Hadid will ever receive a minimum monthly salary of IDR 32,000,000 before he retires (if so at what age will he get it?). He examined the information to establish a pattern using an arithmetic series while designing the solution and then stated that if the sum of the initial age and the n from 32.000.000 = a + (n-1) b was more than 60, then Mr. Hadid will never earn 32.000.000/month.

During the evaluation phase, we concentrated on student activities to provide conclusive responses, construct visualizations based on prior experiences as artifacts, and acquire the essential knowledge. During this step, we attempted to negotiate the meaning of the student-created graphics in the paradigm case (ie, arithmetic series). In a later phase, we focused on confirming the paradigm example and expanding our understanding by drawing linkages to additional bodies of information (such as comparing age, and salary amounts). Throughout the interview, we posed questions designed to elicit cognitive descriptions of the thought processes employed by the participant.

At the problem-solving stage, S2 was able to interpret the information obtained into the arithmetic sequence formula and come to the conclusion that Mr. Hadid did not receive a minimum salary of 32,000,000 because the sum of the n-values was greater than 60. This was done so that he could draw the conclusion that Mr. Hadid did not get a minimum salary of 32,000,000. The student did a second round of analysis on the findings as part of the conclusion process, then he visualized the solution to check if it was correct or if it was still incorrect. S2 emphasized the complexity of the problem while he was checking back on his answer. He admitted that it was difficult to calculate Mr. Hadid's age when he received a salary of 32,000,000.

Students picture the framework that is involved in a given problem solution in an effort to evaluate or test the accuracy of that solution, which is a challenging endeavor (Campbell et al., 1995). According to Egan (2005), the use of one's imagination in the classroom does not lend itself easily to the development of useful methods and procedures that can be utilized by any instructor. This research examines imagination from a pedagogical perspective; the scope of imagination may be unbounded, and its relationship to mathematical material may be difficult to discern. When, on the other hand, the emphasis is placed on visualization and the presentation of open-ended issues, the imaginative process can be observed to be at work within the mathematical material (Wellner, 2022). Based on the results of observations on the student's imaginative processes at the evaluation stage, imagination was formed when the student concluded the final answer by constructing visuals from previous experiences as artifacts and gathering necessary knowledge.

Imaginative process in the creation

Table 4 presents the interview with the third student (S3).

Table 4. The interview with S3

Code	p/j	Interview content
S3me23-001	*P*	*What can you learn from this question?*
S3me23-001	J	*The known pattern is 100000100000 and so on. It means that one is followed by five zeros (0). The numbers are repeated*
S3me23-002	P	*What is asked by the question?*
S3me23-002	J	*The number of 0 to the 100th term*
S3me23-003	P	*How do you solve the problem? What should you do first?*
S3me23-003	J	*This pattern keeps repeating. There are six numbers. I then created an arithmetic sequence with the common difference of 6, so 6 12 18 and so on until I find the 100th term*
S3me23-004	P	*Then?*
S3me23-004	J	*The 100th term must be a number. I believe there is the first term in an arithmetic sequence, so I tried to find the number of "1" until the 102nd term. The number 1 appears 17 times. It means that 100 minus 17 equals 83. There are 83 zeros.*
S3me23-005	P	*Are you sure?*
S3me23-005	J	*Yes. I have made an accurate calculation.*

Notes. S1 (subject 1), me (creation), P (questions), J (answers)

The aforementioned interview excerpts demonstrate that S3 was able to recognize the known aspect of the problem, namely that six numbers are always repeated. In addition, he was able to recognize the question, which was to determine the amount of 0s if the pattern was extended until the hundredth term. S3 devised a novel solution to the problem, which was to identify the number of 0s and 1s, during the period of creation. The most essential aspect of the human creative mind is the breakthrough associated with inventions that transcend limits and established norms. A breakthrough is, on the one hand, an endeavor to identify a new answer or resolution. The concept of imagination, both in common parlance and from a psychological one, is frequently unclear, particularly for the layperson in this field. Therefore, in this context, imagination is understood to be an aspect of the human mind that serves to generate something new (Rodionov, 2013).

S3 solved the problem by counting the numbers 0 and 1 using multiples of 6 to approach 100. The student concluded the sum of the number 1 and subtracted 100 from the number of 1s previously calculated. S3 then double-checked the solution and the method for solving the problem. Based on the results of observations on the student's imaginative process at the creation stage, imagination was formed when the student carried out a cyclical thinking process, where the student thought of new ideas to solve the problem at hand. This process repeatedly occurred until the student decided that no other idea was suitable for solving the problem. Creative students tend to be able to uncover problem-solving ideas for a long time, even so, students are more able to play/turn on their imaginations compared to students in general. This process proves that students are able to design their own way of solving problems, the tendency for students to suddenly shut up and try to repeat the same method indicates that students are activating their imagination (Plsek, 1996).

Conclusion

The results showed that in the *analysis* phase, imagination was found in the students' ability to define problems in general (common visual). As the first step in solving a problem, students corrected the mathematical knowledge needed to define the problem. In the *evaluation* phase, imagination was formed as students completed the final answer by creating visual representations from previous experiences as artifacts taken together and gathering the necessary knowledge. At the *creation* stage, imagination was formed as students engaged in a cyclical thought process, namely, coming up with new ideas to solve their problems. This process repeated itself until the students decided there was no other idea or way to solve the problem. Creative students tend to be able to uncover problem-solving ideas for a long time, even so, students are more able to play/turn on their imaginations compared to students in general (Plsek, 1996).

The finding of this study demonstrates that the capacity for imaginative thinking is essential for studying mathematics since it can enhance students' problem-solving abilities in class. The analysis, evaluation, and creation phases outline the crucial place of imagination in mathematical problem-solving. A teacher is recommended to respond appropriately to each student's response so that they can feel respected and at ease**.**

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