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Research articles

A didactical design for introducing the concepts in algebraic forms using the theory of praxeology

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Abstrak Penelitian fenomenologi ini bertujuan untuk mengkaji secara mendalam pemaknaan siswa terhadap konsep-konsep dalam bentuk aljabar, baik pada proses belajar sebelumnya maupun pada implementasi desain didaktis, serta pengalaman mengajar guru sebagai landasan dalam membuat desain didaktis. Enam siswa kelas 7 dan seorang guru matematika menjadi partisipan pada penelitian ini, yang terdiri dari tiga tahap, yaitu: (1) analisis hambatan belajar siswa melalui uji tes dan wawancara guru, (2) menyiapkan lintasan belajar hipotetik (HLT) siswa dan desain didaktis berdasarkan identifikasi hambatan belajar siswa dan wawancara mendalam dengan guru, dan (3) mengimplementasikan desain didaktis kepada siswa. Hasil penelitian ini menunjukkan bahwa siswa mengalami hambatan didaktis yang disebabkan oleh cara guru mengajar. Guru cenderung memberikan definisi formal pada konsep-konsep bentuk aljabar dilanjutkan dengan contoh-contoh soal. Hal ini mengakibatkan siswa mengalami hambatan yang bersifat epistemologis, dimana pengetahuan siswa terhadap konsep-konsep bentuk aljabar terbatas berdasarkan apa yang dijelaskan oleh guru. Kemudian, dirancang HLT yang menjembatani pengetahuan aritmatika siswa dengan aljabar. Rangkaian tugas disusun berdasarkan teori *praxeology* dengan mengambil konteks kehidupan sehari-hari bertema *Ayo Menabung*. Selama proses pembelajaran, siswa menggunakan berbagai macam representasi seperti simbol dan huruf, untuk menunjukkan variabel, mengemukakan alasan dari penggunaan representasi tertentu, dan mengungkapkan definisi variabel sesuai dengan hasil pekerjaannya. Hal ini juga berlaku pada keempat konsep lainnya. Melalui tugas yang diberikan, siswa dapat berpartisipasi aktif dalam membangun pengetahuan tentang konsep-konsep dalam bentuk aljabar.

Kata kunci *Desain didaktis, Hambatan belajar, Praxeology, Bentuk aljabar*

Abstract This phenomenological study aims to thoroughly investigate the meaning of concepts in algebraic forms developed by students, both in the past learning process and in the didactical design implementation, as well as a teacher's instructional experiences as the basis for preparing a didactical design. Six seventh-grade students and a mathematics teacher participated in the study, comprising three phases: (1) analyzing students' learning obstacles through a test and teacher's interview, (2) preparing hypothetical learning trajectory (HLT) and didactical design based on the identification of the obstacles and in-depth interviews with the teacher, and (3) implementing the didactical design. This study revealed that students have a didactical obstacle because the teacher delivers formal definitions of algebraic form concepts followed by examples of problems. It results in epistemological obstacles, as students' understanding of the concepts is limited according to what the teacher explains. Furthermore, an HLT was developed that bridges students' arithmetic knowledge with algebra. The series of tasks were organized, referring to the theory of praxeology by taking the daily-life context *Let's Save*. During the learning process, students use different representations, such as symbols and letters, to demonstrate the variable, reason for using a particular representation, and state the definition of a variable based on their work. The procedure was also applied to the remaining four concepts. Through the tasks, students can actively construct their conceptual understanding of the concepts in the algebraic forms.

Keywords *Didactical design, Learning obstacles, Praxeology, Algebraic forms*

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Introduction

Several studies (e.g., Jupri et al., 2014) reveal that students continue to view algebra as a complex subject to learn. The challenges encountered by students when learning algebra were due to their transitions from elementary school, which was dominated by arithmetic (Herscovics & Linchevski, 1994). Many students struggle to understand the meaning of algebraic symbols (MacGregor & Stacey, 1997). Furthermore, the concepts in algebraic forms such as variables, coefficients, constants, and terms are the key elements in introducing formal algebra. Nevertheless, related studies revealed that students still have difficulties in defining the algebraic form concepts, such as using variable notations to show a label or object rather than its quantity or replacing the variable with numbers according to its alphabetical order (Adu-Gyamfi et al., 2015; Lucariello et al., 2014; Blanton et al., 2017). Moreover, some students still made errors in distinguishing between variables, coefficients, and constants (Beeh et al., 2018). Students obtained $7n$ as the result of $3 + 4n$ because they did not understand the concept of like and different terms (Khalid et al., 2020). Therefore, it is necessary to design a learning activity that could help students understand the concepts in algebraic forms.

Before developing a learning design, called didactical design, in this study, it is necessary to identify students' learning obstacles. There are three types of learning obstacles: ontogenic, didactical, and epistemological (Brousseau, 1997). A didactical design should contain a hypothetical learning trajectory (HLT) that begins the learning situations with students' prior knowledge to attain the learning goal, which is the construction of new knowledge (Clement & Sarama, 2004; 2009; Gravemeijer, 2004; Suryadi, 2019). In this study, the HLT, along with mathematical tasks, were developed to link students' arithmetic knowledge to algebra in the concepts in algebraic forms. We adopted praxeology by Chevallard (2006; 2019), which comprises task, technique, technology, and theory to study how students build knowledge in the concepts in algebraic form through the problems given. The praxeology has been used to study mathematical tasks, referred to as the epistemological model of mathematics knowledge, both in math textbooks and didactical designs (Wijayanti & Winsløw, 2017; Shinno & Mizoguchi, 2021). Praxeology is also proven to demonstrate students' mathematical activity thoroughly, e.g., students perform diverse techniques and explain accordingly to their chosen technique (Barraza-García et al., 2020). Moreover, the problems that will be assigned to the students are context-based since it can engage and motivate them, reinforces their mathematical concepts, and improves their understanding (Moschkovich, 2002; De Lange, 1996). Thus, taking into account the students' learning obstacles, HLT, and suitable tasks can be the first step in developing an appropriate didactical design.

Previous studies have developed didactical designs with the topic of algebra using real context. The focus is mainly on introducing one-variable linear equations (OVLE), emphasizing shifting the students' operational view to the relational view of equal signs (Nurhasanah et al., 2019). For example, Papadopoulos and Patsiala (2019) applied the Tug-of-War Game, and Saraswati et al. (2016) developed the Algebra Tiles to help students learn OVLE. Otten et al. (2020) promoted physical and pictorial representation to teach linear equations to develop students' algebraic reasoning. In another context, Chase and Abrahamson (2018) elaborated on the use of technology to facilitate students constructing an understanding of how to solve linear equations with the Giant Steps problems. Nevertheless, there have been limited studies concerning developing learning activities with the concepts in algebraic forms. One such study is conducted by Maudy et al. (2017) to introduce the meaning of variables in general, not specific to the context of linear equations, using the context of volume in a test tube. The present study emphasizes the didactical design for the concepts in the algebraic forms. We extend the study by Maudy et al. (2017) by introducing variables, coefficients, constants, and like terms. We argue that understanding the algebraic forms is crucial for students to learn other algebra concepts.

The present study aims to have an in-depth investigation of the students' developed meaning of concepts in the algebra form and the teacher's instructional experiences as the basis for preparing a didactical design. To achieve the purpose, there are three questions addressed in this study: (1) how is the students' learning obstacle when they learn the concepts in algebraic forms? (2) how are the HLT and the didactical design according to both the analysis of students' learning obstacles and in-depth interviews with the teacher related to teaching experiences? (3) how is the implementation of the didactical design in the classroom?

Theoretical Review

The concepts in algebraic forms

In this research, we attempt to study mathematics knowledge in accordance with its scholarly knowledge. According to Chevallard (1989), scholarly knowledge refers to the knowledge— the mathematics knowledge produced by mathematicians. Chevallard argued that the mathematics knowledge from scholars needs to go through a long transformation process so it can 'live' in the school system. Since the knowledge transformation was done by many parties, however, the knowledge learned by students might happen to denature from its original definition (Kang & Kilpatrick, 1992). In response to this, scholarly knowledge will be used to assess whether or not the students' understanding of the concepts in algebraic forms is akin to scholarly knowledge. It can be used to identify students' learning obstacles. Likewise, the scholarly knowledge of the concepts in algebraic forms will serve as the foundation for designing the problems assigned to the students.

In this study, there are four concepts within algebraic forms that will be introduced to the students. The four concepts are variables, coefficients, constants, and like terms. A variable is a symbol representing any value within a designated set of numbers called the domain (Schoenfeld & Arcavi, 1988). Here we use "any value" for variables in the form of algebraic expression as we introduce the term variable to students. In a general context, the term coefficient is used for any number that serves as a measure or characteristic of a set of data (Tanton, 2005). Since we focus on the introduction of algebraic forms, "a set of data" is named as variables. We refer to a coefficient as a constant multiplier of the variables. The term "constants" in algebra refers to a certain number that does not involve any power of the variable (Tanton, 2005; Clapham & Nicholson, 2009). Last, we adopted the study of Linchevski and Herscovics (1996) to define "like terms" as the terms containing the same variable generated to the same power. Here we introduce the student to "like terms" as the idea of grouping like terms, which first gives the literal symbol an operational dimension (Linchevski & Herscovics, 1996).

Learning obstacles

A learning obstacle is a condition that hinders the acquisition of new knowledge by students throughout the learning process, thus possibly making them experience learning difficulties. This difficulty is reflected in students' errors, for example, when they were asked to solve problems after they learned a certain mathematics concept (Brousseau, 1997; Suryadi, 2019). Furthermore, Kansanen and Meri (1999) explain that learning obstacles can be investigated based on the triadic relationship between teacher-student-subject/mathematics content. According to its source, learning obstacles are categorized into ontogenic, didactical, and epistemological.

An ontogenic obstacle is a learning obstacle that appears in relation to the student's development of his intellectual ability. This learning obstacle may result from an imbalance between the learning activity or the knowledge that would be constructed by the student with his intellectual readiness or ability level (Brousseau, 1997). For example, if the problem given is considered too complicated by the student, then he will face difficulties in solving it. Otherwise, the student has no motivation to solve the problem if it is considered too easy.

A didactical obstacle is a learning obstacle associated with the stages of teaching material implemented by the teacher. According to Suryadi (2019), the stages of teaching material that are not detailed and too detailed can influence the student's learning process. Thus, the didactical obstacle can be identified from the didactical design made and implemented by the teacher in the classroom. Additionally, the obstacle also appears if teachers only focus on the goal of the study, for example, '*what matters are students know the formula and can solve the problem*'.

An epistemological obstacle is a learning obstacle related to how students understand the knowledge. In this study, it is mathematics knowledge. The obstacle is recognized when there is a limitedness in students' understanding of knowledge only in a particular context (Brousseau, 1997). It is possible to identify epistemological obstacles when a student's understanding of a mathematics object works well for a certain problem but is inappropriate for the other.

In this study, learning obstacles will be analyzed from a test about the concepts in algebraic forms that will be given to the students and an interview with the teacher.

Hypothetical learning trajectory (HLT)

HLT is a variety of predictions of learning situations containing three components: learning objectives, student learning, and thinking processes, and learning activities that will engage students during the learning process (Clements & Sarama, 2004; 2009; Gravemeijer, 2004). It is responsible for providing a path for students to learn new knowledge (Andrews-Larson et al., 2017). HLT begins by linking learning situations that employ students' prior knowledge to achieve the learning goals, which is the development of new knowledge by students. In elaborating our HLT, we consider an HLT to include three interrelated aspects: (1) learning objectives about introducing the concepts in algebraic forms; (2) the development of students' mathematical activity (from arithmetic to algebra); and (3) the series of tasks that will engage the students during the learning activity.

Praxeology

Praxeology is considered the main idea of the anthropological theory of didactics (ATD), where Chevallard (2006) conveys that it is the theory for a thorough analysis of human actions and behaviors. Praxeology proposes that initially, there is no action or behavior performed by an individual without reason for him to carry out such action or behavior (Chevallard et al., 2015). Moreover, praxeology comprises two interrelated components: *praxis* and *logos*. *Praxis* (practical block) is defined as a human activity, and *logos* (knowledge block) refers to human reasoning (Chevallard, 2006). *Praxis* is formed by a *type of task* (T)—the problem or situation given and the *technique* (τ) to solve the problem. Moreover, *praxis* always requires *logos* that consists of *technology* (θ) to give a reason for the *technique* used and *theory* (Θ) to justify the *technology* (Chevallard, 2006; 2019). Figure 1 shows how the position of the four components in praxeology plays their role.

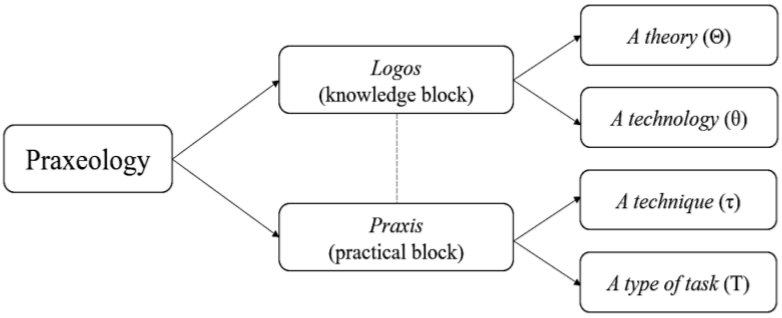


Figure 1. Four components of praxeology (adopted from Chevallard, 2006; Putra & Witri, 2017)

In mathematics learning, praxeology can serve as a guide for creating a series of tasks that will be given to students to assist them in constructing knowledge about certain mathematics objects. Normally, a *task* can be solved with various *techniques*, as well as *technology* can be used to justify some *techniques* (Putra & Witri, 2017). In the end, the *technology* will lead the students to build new mathematics objects (*theory*). Therefore, Table 1 summarizes the use of praxeology in designing tasks to introduce the concepts in algebraic forms.

Table 1. Designing tasks to introduce the concepts in algebraic forms based on praxeology

Praxis		***Logos***	
Type of task (T)	*Technique* (τ)	*Technology* (θ)	*Theory* (Θ)
Problems about the concepts in algebraic forms that will be given to the students. The problems will be designed to have more than one solution.	Students' way of solving the problems. Initially, students will generate different answers as they explore the *technique* by themselves.	Argumentation, explanation, and justification were used by students regarding their *technique*.	Students will generate meaning from the concepts in algebraic forms as a result of good argumentation in the *technology* phase. In this phase, students' knowledge of variables, coefficients, constants, like terms, and unlike terms is separated from the context of the problem (abstract).

Methods

This study employed a phenomenology approach (Creswell, 2016) since it aims to interpret and describe the meaning of concepts in algebraic forms constructed by students through their learning experiences in the class and the teacher's teaching experiences. The students' experiences were divided into two parts in order: (1) exploring students' prior learning experiences for the concepts in algebraic forms to identify their learning obstacles and (2) exploring the students' experiences when learning the concepts in algebraic forms throughout the implementation of the didactical design.

Participants

The participants in this study were six seventh graders and a mathematics teacher. The students were chosen since algebraic forms were introduced at this level, particularly during the first semester. At the time this study was conducted, the students were already in the second semester, so we were able to identify their learning obstacles in the concepts in algebraic forms. Initially, we planned to have a full-class student (21 students). However, since the research was conducted during the pandemic and still in online-learning mode, not all students were willing to fill out the test. There were only six students submitted the answer. Likewise, the teacher was selected since only one teacher teaches mathematics in grade 7 in the participated school.

Instrument

Aside from the researchers, the instruments included a test and a guideline for the teachers' interview. The test consisted of six problems assigned to the students to assess how is their understanding of the concepts in algebraic forms. The problems were divided into three parts (Table 2): Part 1 was intended to reveal student's definition of each concept in algebraic forms and will be compared to the concepts produced by mathematicians; Part 2 was intended to assess how students employ their understanding of algebraic form concepts, into algebraic expression problems; and Part 3 was intended to assess how students operate their understanding of algebraic form concepts into contextual problems. The guideline for teacher's interviews was made as we wanted to explore more about how the learning process was. The interview questions were elaborated on three categories: (1) how the teacher designed the learning activities, (2) how the learning design was implemented, and (3) how the student's conceptual understanding of the algebraic forms as a result of the learning process was.

Table 2. The test about the concepts in algebraic forms

Part 1: Students' definition of each concept in algebraic forms
1. With your own definition, explain what the meaning of variables, coefficients, and constants is! 2. With your own definition, explain what is like terms in algebra!
Part 2: How students employ their understanding of the concepts in mathematics problems
3. Determine the variables, coefficients, and constants from the following algebraic forms: a. $4x + 1$ b. $-2p + 5pq + q$ c. $x^2 - 7x + 3z + 4$ 4. Determine the like terms from the following algebraic forms: a. $2x + 3y + 5x$ b. $a^2 + 4b + a - 4$ c. $xy + 6x$ d. $p - 8q^2 + 3p^2 + q^2$
Part 3: How students employ their understanding of the concepts in contextual problems (adapted from Khalid et al., 2020)
5. In a minimarket, each chocolate prices for c rupiah and each candy prices for p rupiah. If a person buys 3 chocolates and 4 candies, then what is the meaning of $3c + 4p$? 6. In a bike shop, there are x mountain bikes and y toddler bikes. If each mountain bike has 2 wheels and each toddler bike has 3 wheels, then what is the meaning of $2x + 3y$?

Data collection and analysis

Overall, this study was carried out in eight stages as follows.

1. The focus of the research was established.
2. The instruments (a test and guidelines of the teacher's interviews) were developed.
3. The criteria of participants were determined; mathematics teacher and six seven-graders from one school.
4. A 30-minute written test was administered to the students using *Google Form*. The test's results were to provide information on the students' prior knowledge of the concepts in algebraic forms. Afterward, a recorded teacher's interview was done for approximately 30 minutes by *Zoom* application. It was held to attain further information about how the teacher planned her didactical design in the topic and how its implementation in the classroom.
5. Students' answers to the test were analyzed by classifying their responses into two categories: (a) their responses indicate similar concepts to the scholarly knowledge and (b) their responses represent significantly different concepts to the scholarly knowledge. A comprehensive analysis of both students' answers (second category) and teacher's interview results was done based on three types of students' learning obstacles by Brousseau (1997): (a) didactical obstacles, if there is an issue related to the way teacher presents the material, (b) epistemological obstacles, if the algebraic form concepts taught by the teacher are likely different from the concepts produced by mathematicians; and (c) ontogenic obstacles if the tasks provided by the teacher are incompatible with their existing knowledge.
6. To obtain detailed information on the teacher's teaching experiences, an in-depth interview with the teacher was carried. The interview result and descriptions of students' learning obstacles will be used to construct HLT and the didactical design. In this phase, praxeology by Chevallard (2006; 2019) served as the foundation for developing the series of tasks to introduce the concepts in algebraic forms. The final didactical design was validated using investigator triangulation by a mathematics educator, mathematics educationalist, and mathematician.
7. The didactical design was implemented on the students for 3×30 minutes (one meeting). Moreover, the learning design was implemented by one of us (the researcher) through video conference *Zoom*. Additionally, we referred to the *praxeology*, a theory developed by Chevallard, throughout the learning process.
8. The results of implementation will be evaluated and reflected according to the praxeology theory. In this case, the implementation of the didactical design is said to be successful if all components in praxeology were satisfied during the learning process and the students construct concepts in algebraic forms similar to those generated by mathematicians.

Findings and Discussion

The findings of the present study are discussed in three main stages: the analysis of students' obstacles in learning the concepts in algebraic forms, the development of HLT and didactical design to introduce the students to the concepts in algebraic forms, and the implementation of didactical design in the classroom.

Students' obstacles in learning the concepts in algebraic forms

To obtain information about how students' understanding of the topic, a diagnostics test was assigned. The test was divided into three sections. The first section is about students' definitions of the five concepts in algebraic forms. Students' definitions of variables, coefficients, constants, and like terms vary. When compared to the definition of variables produced by mathematicians as "*a symbol that can represent any value within a designated set of numbers called the domain*" (Schoenfeld & Arcavi, 1988), none of the students defined variables according to the scholarly definition. Most of them focus on three terms: symbol, letter, and unknown. Similarly, the students' conceptual understanding of coefficients also differs from what is defined by mathematicians. They typically write "*a number in front of variables*" instead of "*a constant multiplier of variables*." Their understanding of the term constants closely resembles its original definition, "*a certain number that does not involve any power of the variable"* (Tanton, 2005; Clapham & Nicholson, 2009). Nevertheless, the students' definition stressed the position of constants since it is usually placed at the end of an algebraic expression. Lastly, most students have a similar definition of like terms like its scholarly knowledge, as "*the terms containing the same variable generated to the same power*" (Linchevski & Herscovics, 1996), but one student asserted just "*the same variable*."

In the second section of the test, we tried to investigate how students applied their conceptual understanding of mathematical problems. We assigned three problems and asked the students to determine the variables, coefficients, and constants (see Table 2). Based on their answers, all of them wrote the variables and constants in each algebraic form correctly. However, they wrote incorrect answers for coefficients starting from number 3b. For example, in the form of $-2p + 5pq + q$, students only wrote 2 and 5 as the coefficients. The answers were expected since they defined coefficients as merely a number in front of a variable. Furthermore, there are four problems that asked the students to determine the like terms. Figure 2 shows an example of the students' answers in determining the like terms. We can see that there are students who only focus on the same variables but not their degrees or the same numbers. The results are similar to Beeh et al. (2018) in that students refer like terms as terms with identical letters but ignore their degree.

Determine the like terms from the following algebraic form:

$$a^2 + 4b + a - 4$$

Students' original answers	Translation
Tidak ada, karena tidak ada variabelnya yang sama	None, because there is no same variable
Tidak ada	None
Suku sejenis=a^2 dan a	Like terms=a^2 dan a
$a^2 + a + 4b - 4$	$a^2 + a + 4b - 4$
$4b$ dan 4	$4b$ and 4
$4b, -4$	$4b, -4$

Figure 2. Students' answers in determining like terms

In the third section of the test, we investigated how students applied their conceptual understanding in contextual problems, particularly in the concept of variables. There are two problems given to the students (Table 2). As we analyzed their answers, they still made errors in translating the given algebraic forms according to the problem's context. Figure 3 depicts the students' answers, where only one student almost got the answer correct. The correct answer is "the total price of 3 chocolates and 4 candies," while he wrote $3c$ as the price for 3 chocolates and $4p$ as the price for 4 candies separately. The rest of the students got incorrect answers, as half of them defined variables c and p as the objects. The students' interpretation of variables is similar to previous studies (Küchemann, 1978; Akhtar & Steinle, 2017; Khalid et al., 2020), where students associated variables with the object/letter rather than the quantity.

To summarize, we classified students' written work into two categories. First, some students have a good understanding of the concepts of algebraic forms. It can be seen that students define the term constant correctly and identify the like terms from the given algebraic forms correctly. Second, other students have a less conceptual understanding of algebraic form. It is indicated by how students define the variable as a symbol or an abbreviation letter of an object (e.g., c for chocolate), the coefficient as the number in front of the variable, and like terms as terms with the same number or letter. We predicted that students in the second category have learning obstacles.

In a minimarket, each chocolate prices for c rupiah and each candy prices for p rupiah. If a person buys 3 chocolates and 4 candies, then what is the meaning of $3c + 4p$?

Students' original answers	Translation
Yaitu 3c adalah harga dari 3 coklat,3×Harga/1 coklat. 4p adalah harga dari 4 permen yaitu 4 ×Harga/1 permen	$3c$ is the price for 3 chocolates and $4p$ is the price for 4 candies
3 coklat + 4 permen	3 chocolates + 4 candies
3 coklat 4 permen	3 chocolates 4 candies
3c = menggambarkan harganya yaitu c dan jumlahnya ada tiga. 4p = menggambarkan harga satu permen yaitu p, dengan jumlah 4 permen	$3c$ = c illustrates the price and 3 is the number. $4p$ = p illustrates the price of candy with the number of candy is 4
3c + 4p adalah bentuk penjumlahan aljabar dari 3 coklat dan 4 permen. 3c=3 coklat dan 4p=4 permen	$3c$ = 3 chocolates and $4p$ = 4 candies
Jadi c dan p itu mewakili harga masing2 barang,karena seseorang tersebut membeli 3 coklat dan 4 permen,maka jumlah barang dikali dengan harga barang tersebut,maka bisa disimpulkan c dan p adalah variabel, 3 dan 4 adalah koefisien	c and p represent the price for each stuff. Because a person buys 3 chocolates and 4 candies, then the number of stuff multiply by the stuff's price. It can be concluded that c and p are variables, 3 and 4 are coefficients.

Figure 3. Students' answers in translating the mathematics model into the problem's context

Furthermore, we sought to explore more about possible factors that influence students' conceptions of variables, coefficients, constants, and like terms in algebra. We conducted an interview with the teacher to obtain information about how was the implementation of teaching and learning the algebraic forms. The following discourse is a part of the teacher's interview (Transcript 1), where we coded R for the researcher and T for the teacher. The number after each code indicates the order of question (for the researcher) and response (for teacher), e.g., $R1$ means researcher in question 1.

Transcript 1

$R1$ *: Could you explain to me how you designed and implemented the learning activity in the concepts in the algebraic forms, such as variables, coefficients, and so on?*

$T1$ *: Actually, I am a new math teacher here and started to teach in the second semester while algebraic form was learned by students in the first semester. But I got to teach them the linear equation with one variable.*

$R2$ *: Ok, I assume to learn the linear equation with one variable, students must understand first the meaning of variables, coefficients, and so forth, right?*

$T2$	*: That is the problem. There were many students who did not understand those concepts, so I could not proceed to the linear equation with one variable directly.*
$R3$	*: Do you know why they do not understand?*
$T3$	*: As far as I know, the former teacher is actually a history teacher, but he had to cover math since, at that time, the school did not have enough mathematics teachers. So, you know, with no background study in math, he had limitedness in teaching mathematics concepts.*
$R4$	*: Do you perhaps know how he teaches?*
$T4$	*: According to what my students said, the teacher only gave the definition of variables, coefficients, and so forth, then continued to the example of each term, like x is variables, 2 in $2x$ is called coefficients.*

We underline the teacher's final remark (Transcript 1, $T4$) regarding the former teacher by simply giving formal definitions of each concept followed by examples. We infer that the didactical design made and implemented by the former mathematics teacher did not adequately assist students in conceptualizing variables, coefficients, constants, and like terms. According to Brousseau (1997), *the students have a didactical obstacle* since their difficulties in understanding the concepts in an algebraic form are derived from the teaching material implemented by the teacher. Additionally, as students only received formal definitions for each concept (variables, coefficients, constants, and like terms) and "examples," their knowledge of the concepts is confined to what the teacher has given. In this case, *the students also have an epistemological obstacle*. As an illustration, students interpret that coefficient as a number in front of the variable because the teacher gave an example, "2 in $2x$ is coefficient".

Hypothetical learning trajectory and didactical design in the concept of algebraic forms

We interviewed the teacher in-depth to acquire information about the teacher's teaching experiences and the students' characteristics. The following conversation is from the teacher interview (Transcript 2), where we labeled it R for the researcher and T for the teacher. For instance, $R1$ denotes the researcher in question 1 and $T1$ for the teacher in response 1.

Transcript 2

$R1$	*: How is the students' characteristic that will be participating in this research?*
$T1$	*: Students in grade 7A have an excellent mathematical understanding and are quite responsive during the learning process. However, it is better to give students tasks or questions related to daily-life application, particularly in algebra. It is because students learned abstract symbols, except numbers, in algebra for the first time.*
$R2$	*: How many meetings are needed for learning the concepts in algebraic forms?*
$T2$	*: Usually, once is necessary for introducing the variables, coefficients, constants, and terms. There are two meetings once a week for math, 3×30 minutes and 2×30 minutes for introducing the variables and so forth, 3×30 minutes is enough.*
$R3$	*: Do you have any recommendations for creating the tasks so they can be suitable for the students?*
$T3$	*: Like I mentioned before, students need to understand how each concept is applied in daily-life problems, not merely in the form of algebraic expression. Also, it is better for students to learn each concept gradually. For example, they learn about variables, then coefficients, then terms.*
$R4$	*: Is there any suggestion again?*

$T4$: *Since the tasks that will be developed in the form of contextual problems, do not just present the tasks in narration only, if possible. Pictures and other illustrations might be added to the problem context. The students are sometimes confused if they have to read a long narration.*

According to Transcript 2 ($T1$), the teacher advises that contextual problems are preferably used in the series of tasks that will be developed. The suggestion is in line with the identification of students' learning obstacles, the former teacher's approach to teaching algebraic forms looks heavy towards the formal way. It caused students to become less active in constructing meaning. Subsequently, Transcript 2 ($T3$) proposes that students need to learn each concept in algebraic form systematically. Therefore, it is recommended to begin the learning with arithmetic, a concept that already exists in students' knowledge, and integrate arithmetic into the series of tasks, until it is transformed into new knowledge that is the concepts in algebraic forms. The results in students' learning obstacles and the teacher's interview altogether serve as the foundation for creating the HLT and didactical design. The HLT we have developed is shown in Figure 4.

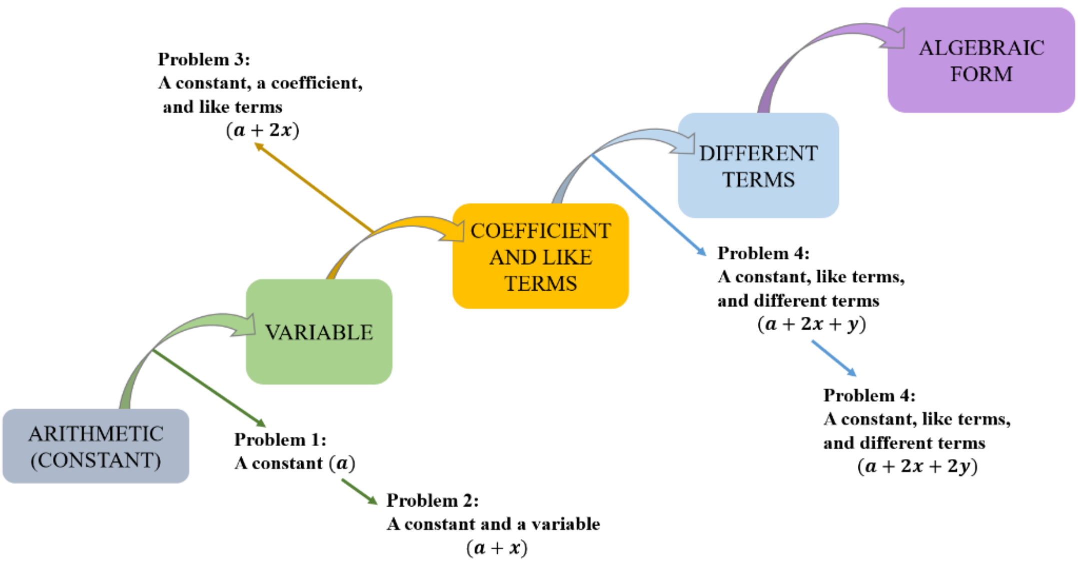


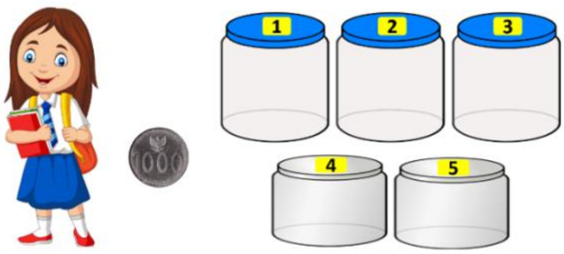
Figure 4. Hypothetical learning trajectory for introducing the concepts in algebraic forms

There are three aspects included in the HLT:

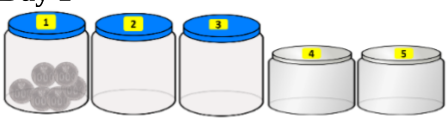
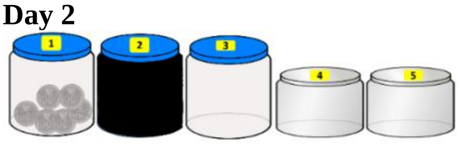
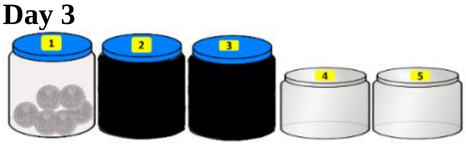
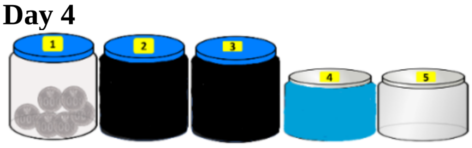
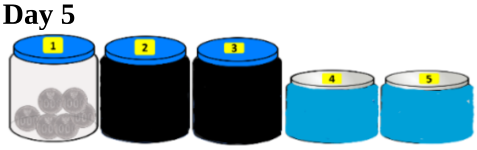
1. Learning objective, that is, students can construct the meaning of variable, coefficient, constant, and like terms in algebra.
2. The development of students' mathematical activities. The learning begins with students' knowledge of arithmetic (in the form of a constant), then integrates with the use of the variable, then the use of coefficients and like terms. At the end of doing the five problems given (see Figure 4), it is hoped that students build new knowledge about variables, coefficients, constants, and like terms.
3. The series of tasks that will be included in the didactical design. Based on Figure 4, we designed five problems to assist the student in constructing the meaning of algebraic forms. From the analysis of learning obstacles, the students were not exposed to the use of the concepts in contextual problems since they were given the concept of each term instantly. Thus, we created contextual problems with the theme *Let's Save* (Table 3).

We also considered the teacher's suggestion from Transcript ($T4$) to feature pictorial representations in the task series, as shown in Table 3.

Table 3. The problems designed with the theme "Let's Save"



Anisa has 5 empty jars. All of the jars will be used as a place for her to save money in the form of Rp1000,00 coins. Of all jars, Jar 1, 2, and 3 have the same shape and size, while Jar 4 and 5 have the same shape and size.

Day 1 	On the first day, Anisa fills Jar 1 with Rp1000,00 coins, as shown in the picture. How much money is saved by Anisa on this first day?
Day 2 	On the second day, Anisa decides to cover Jar 2 with a black paper so that we cannot see the amount of money in that Jar. Later, Anisa saves some money with Rp1000,00 coins in Jar 2. What is the total money saved by Anisa until this second day?
Day 3 	On the third day, Anisa decides to cover Jar 3 with a black paper as she did on Jar 2. Later, Anisa saves some money with Rp1000,00 coins in Jar 3, as much as she saved in Jar 2. What is the total money saved by Anisa until this third day?
Day 4 	On the fourth day, Anisa decides to cover Jar 4 with a blue paper so that we cannot see the amount of money in that jar. Later, Anisa saves some money with Rp1000,00 coins in Jar 4. What is the total money saved by Anisa until this fourth day?
Day 5 	On the fifth day, Anisa decides to cover Jar 5 with a blue paper as she did on Jar 4. Later, Anisa saves some money with Rp1000,00 coins in Jar 5, as much as she saved in Jar 4. What is the total money saved by Anisa until this fifth day?

According to the earlier study, students are exposed to tasks that begin with the form of a, $a + 3x$, then $a + 3x + 2y$, where a is a constant and x, y are variables (Maudy et al., 2017). We proposed to elaborate on Maudy's HLT by starting the task with a, $a + x$, $a + 2x$, $a + 2x + y$, then $a + 2x + 2y$ (Figure 4) We also prepared the prediction of students' responses and anticipation when working on the tasks. We predicted the students' responses with reference to the components of praxeology (Chevallard, 2006). The *techniques* that students might use, the *technologies* to explain the *techniques*, and *the theory*, which is the concept of algebraic forms. We use the concept of variables by Schoenfeld and Arcavi (1988), coefficients by Tanton (2005), constants by Tanton (2005) and Clapham and Nicholson (2009), and like terms by Linchevski and Herscovics (1996). In the final step of developing a didactical design, a validation of the design was done by investigator triangulation (Flick, 2018). The investigators involved to validate the didactical design are mathematics educator (teacher), mathematics educationalist (mathematics education professors), and mathematicians (mathematics professor).

The implementation of didactical design in the concepts in algebraic forms

We implemented the design through online learning using video conference *Zoom*. All problems or materials were presented by PowerPoint and a pen tablet to make it more interactive. The learning activities were held through four phases as follows:

1. We informed the students about the topic that would be learned as well as the learning goals.
2. In this phase, we started to explain the problem context to be discussed in today's meeting. The problem served, "Let's Save," presented a school girl named Anisa, five empty jars, and a coin with Rp1000,00 written on it. In this part, we also told the students about the rules to solve the next questions.
3. The students were told to solve the 1st question (Day-1 in Table 4) until the 5th question (Day-5 in Table 4). We summarize students' ways and explanations of solving each problem in the following section.

Day 1 problem

All students could answer the problem correctly. The total money saved by Anisa on the first day is Rp6000,00. They multiplied the number of coins in Jar 1 (6 coins) with Rp1000,00 to get the answer.

Day 2 problem

Students tried to use various ways to represent the amount of money saved by Anisa in Jar 2. Some students used dots (…..), question marks (?), and letters (x, k) so that the total money saved until the second day was presented by $6000 +$….., or $6000+?$ or $6000 + x$, $6000 + k$. Moreover, students were asked to explain the reason behind their representation. Students who used (…..) and (?) stated, "*we have no idea how much money in Jar 2, so we used that symbol*" students who used "x" argued that "*x for the amount of money in Jar 2*", and a student with "k" explained "*k for coin* (*koin* in Indonesian), *so it is the amount of money in Jar 2*". Next, we validated that in the context of algebra, the answer x or k was the precise answer to represent quantities whose value are not yet known.

Day 3 problem

After students understood about variable from Day 2, albeit the term 'variable' is not yet told, they used the letter to show the total money saved by Anisa. In the beginning, students seemed confused to understand the sentence "*Anisa saves some money with Rp1000,00 coins in the Jar 3 as much as she saved in Jar 2*," so we gave a little help until they were able to conclude that the total money in the Jar 2 = the total money in the Jar 3. To show the money saved by Anisa until the third day, there are two representations applied by students: (1) $6000 + x + y$

and (2) $6000 + 2x$. Students with (1) form said, "*because the jars are different, then the letters are also different*," while students with (2) form stated, "*because the money in the Jar 2 equals Jar 3, then the letters are the same, x and x, added become $2x$*". At this stage, we then confirmed that (2) representation was the correct answer because the letter represented the amount of money, not the Jar.

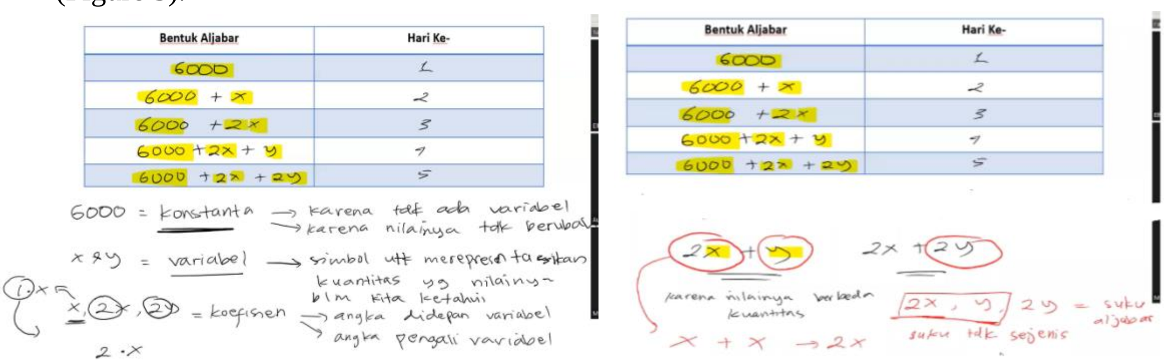
Day 4 problem

Students got more understanding that the same letters are used to represent the same amount of money, even though it was saved in different jars. On the fourth problem, all students answered $6000 + 2x + y$. They used y for the total money in Jar 4 since there was no information if the money in Jar 4 is the same as in Jar 3 or Jar 2.

Day 5 problem

On the last problem, all students answered $6000 + 2x + 2y$. They understood that the amount of money in Jar 2 is the same as in Jar 3, and the amount of money in Jar 4 is the same as in Jar 5.

4. After finishing all of the questions, we asked the students to state the algebraic forms (the representation of the total money saved by Anisa) from each day in the table as follows (Figure 5).



(a) (b)

Figure 5. Explanation about the concept of constant, variable, and coefficient (a), like terms and different terms (b)

Figure 5a presents the algebraic from each problem written by students. We highlighted 6000 from each algebraic form and confirmed that it is called a constant. We also asked the students why 6000 is a constant. Students said, "*because there is no variable*" and "*the value is not changed*." It seems that students understand the concept of constant. Next, we focused on the part x and y and told them that the letters are called variables. The students also explained that variables as "*the symbol to replace the unknown quantities*," which we made clear as "*the symbol to represent the quantities whose value are not yet known*." Moreover, the students defined coefficients as "*the number in front of the variable*." However, we emphasized the meaning of $2x$ according to the problems solved. Some students answered "*2 times x*" or "*there are two x*," so we confirmed that the coefficient is "*the multiplier of a variable or the number multiplied by the variable*." Similarly, in Figure 5b, students explicated that x and x or y and y are the same variables, so they can be combined as $2x$ (the result of $x + x$) and $2y$ (the result of $y + y$), while x and y cannot be combined. In addition, we validated that the former is defined as like terms and the latter as different terms. We can do addition and subtraction to the like terms but not with different terms.

Furthermore, we summarized all students' learning activities according to the four components of praxeology (Chevallard, 2006) in Table 4.

Table 4. Students' learning activities according to the praxeology

***Type of task* (T)**	***Technique* (τ)**	***Technology* (θ)**	***Theory* (Θ)**
T_1: Representing a solution with a constant (a)	τ_1: All students use a number (6000).	θ_1: there is no variable. The value remains the same.	Θ_1: Constant represents the unchanged value.
T_2: Representing a solution with a constant and a variable $(a + x)$	$\tau_{2,1}$: Using a number and a symbol (6000 +….. or 6000+?).	$\theta_{2,1}$: the symbol used to show value or quantity that is not yet known.	Θ_2: A variable is a symbol to represent quantities whose value is not yet known.
	$\tau_{2,2}$: Using a number and a letter $(6000 + x$ or $6000 + k)$.	$\theta_{2,2}$: the letter used to replace the value of a quantity that is not yet known.	
T_3: Representing a solution with a constant, a coefficient, and like terms $(a + 2x)$	$\tau_{3,1}$: Using a number and two different letters $(6000 + x + y)$.	$\theta_{3,1}$: letters are different because the jars are different.	Θ_3: Coefficient is a multiplier of a variable or the number multiplied by the variable. Like terms are terms in algebraic forms that have the same variables raised to the same degree.
	$\tau_{3,2}$: Using a number and two same letters $(6000 + 2x)$	$\theta_{3,2}$: letters are the same because the money in Jar 2 and 3 are the same. The number in front of the variable is the coefficient.	
T_4: Representing a solution with a constant, like terms, and different terms. $T_{4,1}$: $(a + 2x + y)$	$\tau_{4,1}$: Using a number and two same letters, and two different letters $(6000 + 2x + y)$	$\theta_{4,1}$: letters are the same because the money in Jar 2 and 3 are the same. Letters are different because the money in Jar 4 is different from Jar 2 or Jar 3.	Θ_4: Different terms are terms in the algebraic form that have different variables or the same variables but are raised to a different degree
$T_{4,2}$: $(a + 2x + 2y)$	$\tau_{4,1}$: Using a number and two pairs of the same letters, and two different letters $(6000 + 2x + 2y)$	$\theta_{4,1}$: letters are the same because the money in Jar 2 and 3 are the same. Also, the money in Jar 4 and Jar 5. Letters are different because the money in Jar 4 or Jar 5 differs from Jar 2 or Jar 3.	

It can be seen from Table 4 that in several types of tasks, the students used more than one *technique* along with each *technology*. This is per what Chevallard defined: there will be the possibility of various *techniques* used to solve one problem where each *technique* may have its own *technology*/justification. Finally, in the *theory* section, we try to validate students' statements about constants, variables, coefficients, like terms, and different terms as similar to the scholar definition (Schoenfeld & Arcavi, 1988; Tanton, 2005; Clapham & Nicholson, 2009; Linchevski & Herscovics, 1996).

It is apparent from the implementation that students can actively participate in any given problem until they properly construct each concept. An earlier study (e.g., Maudy et al., 2017) revealed a similar outcome in which students employed various representations to show variables. However, there is a lack of information in the former study about students' reasons for selecting a particular representation of a variable, whereas the use of praxeology for didactical design implementation in this study provides a distinct explanation of the variable's representation from each student (*technology* part in the praxeology). Students also learned that letters in variables denote the value of the coin kept in each Jar rather than the coin itself, e.g., x for the amount of money in the Jar rather than x for the coin. We focus more on the concept of a variable because the test results show that some students still see the variable as the object rather than the value or quantity of the object (Küchemann, 1978; Khalid et al., 2020).

Conclusion

In this study, prior to developing and implementing a didactical design to introduce the concepts in algebraic forms. We conducted a prior investigation about students' learning obstacles in the concepts in the algebraic forms. The concepts are variables, coefficients, constants, and like terms. Our findings show that students have didactical obstacles due to the teacher's teaching material. The teacher only delivered the definition of variable, coefficient, constant, and like term then, followed by a mathematical example. Consequently, students are constrained in applying the understanding to contextual problems. Since students' knowledge of algebraic form concepts is limited to what their teacher explains, the didactical obstacle creates an epistemological obstacle.

We also analyzed the teacher's teaching experiences, which gave suggestions for creating the HLT and the series of tasks in didactical design, along with identifying students' learning obstacles. The HLT begins with students' knowledge about arithmetic and integrates within five interrelated contextual problems. Until the end of learning, students can build a conceptual understanding of variables, coefficients, constants, and like terms. Lastly, in the implementation of didactical design, it is shown that students can engage actively to solve each problem. According to praxeology, students use *various techniques*, especially when they demonstrate the variables (using symbol or letter), they also state their own explanation (*technology*) of using such *technique*, and finally, they can construct the meaning of each concept, validated by the researchers (*theory*). At the end of the learning, we reflected that students' conceptual understanding of algebraic forms is akin to the algebraic form concepts provided by mathematicians.

There are several limitations of this study. The analysis of learning obstacles was generated from students' answers to the test and the teacher's interviews. To attain deeper analysis, a further investigation can be done, such as teachers' lesson plans and mathematics textbooks analysis. Additionally, the series of tasks developed only includes the addition operation. Further studies might extend the tasks to include other operations as well.

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