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Research articles

How do secondary students develop the meaning of fractions? A hermeneutic phenomenological study

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Abstrak Memahami makna pecahan merupakan dasar untuk mempelajari topik-topik terkait pecahan. Namun, siswa masih mengalami kesulitan dalam topik penting ini. Penelitian fenomenologi hermeneutika ini bertujuan untuk mengkaji lebih lanjut makna pecahan yang dibangun oleh siswa sekolah menengah, proses mengkonstruksi makna menggunakan berbagai model, dan faktor-faktor yang menghambat pengembangan makna pecahan. Penelitian ini melibatkan dua puluh dua siswa yang diberikan tugas matematika dan beberapa siswa dari berbagai kategori kemampuan matematika dipilih untuk diwawancarai. Analisis tematik menggunakan NVivo-12 menemukan bahwa beberapa siswa memaknai pecahan sebagai alat bantu dan bilangan bulat. Sebagian besar siswa pada semua kategori kemampuan matematika cenderung memaknai pecahan sebagai rasio dan hasil bagi. Namun, mereka tidak bisa menggunakan model untuk merepresentasikan makna pecahan tersebut. Penelitian ini menunjukkan bahwa makna pecahan yang dimiliki oleh siswa terbatas dan berbeda dari makna pecahan yang dimiliki siswa sekolah dasar. Selain itu, terdapat inkonsistensi antara makna pecahan yang diungkap siswa dengan penggunaan model. Keterbatasan dan inkonsistensi tersebut dianggap dipengaruhi oleh kurangnya pengetahuan konten guru tentang materi dan kurangnya penggunaan model dalam mengajar pecahan. Dalam hal ini, pendidikan guru (matematika) harus lebih memperhatikan makna pecahan dan penggunaan model pecahan dalam pembelajaran.

Kata kunci *Fenomenologi hermeneutika, Makna pecahan, Siswa SMP, Model*

Abstract Understanding various meanings of fractions is a foundation to learn related fractions topics. However, students are struggling with this essential topic. This hermeneutic phenomenological study aims to further investigate the meaning of fractions developed by secondary school students, the process of constructing the meaning using models, and factors that hinder the development of the meaning. It involved twenty-two students given fractions tasks, and some students of the different categories of mathematics ability were selected to be interviewed. A thematic analysis using NVivo-12 found that some students interpret fractions as a tool and integers. Most of the students in all categories tend to espouse fractions as ratio and quotient. However, they cannot use the models to represent the meanings. This study reveals that the students' developed meanings of fractions are limited and different from what primary students have. In addition, there is an inconsistency between the espoused meanings and the use of models. The limitation and inconsistency are considered to be affected by teachers' lack of content knowledge on the topic and less attention to using models in teaching fractions. This study implies that (mathematics) teachers education should pay more attention to the meaning of fractions and their related models in learning.

Keywords *Hermeneutic phenomenology, Fractions meaning, Secondary students, Models*

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Introduction

Previous studies revealed that fraction is a fundamental mathematical concept (Cortina, Visnovska, & Zuniga, 2014; Getenet & Callingham, 2021; Kullberg & Runesson, 2013). However, several studies show that it is one of the ‘problematic’ concepts for elementary and middle school students (Adu-gyamfi, Schwartz, Sinicrope, & Bosse, 2019; Harvey, 2012; Zhang, Clements, & Ellerton, 2014). There are several aspects that influence students’ understanding when learning fractions, such as students’ knowledge of integers (Sun, 2019), mathematics tasks given by teachers (Wahyu, Kuzu, Subarinah, Ratnasari, & Mahfudy, 2020), learning trajectories (Confrey & Maloney, 2015), teacher knowledge (Adu-gyamfi et al., 2019; Lee & Lee, 2021), and the complexity of the fraction itself (Li, Chen, & An, 2009; Obersteiner & Tumpek, 2016).

Regarding the meaning of fractions, some researchers examine them using various methods. Martinez and Blanco (2021) provided a problem-posing task, and the results were analyzed quantitatively to determine the types of the meaning of fractions possessed by elementary school students. The study found several meanings, namely part-whole and part-set, because the two meanings are first taught in schools. Gabriel et al. (2013) also employed a quantitative method to determine the meaning of fractions in elementary school students. The study referred to the notion of conceptual understanding to indicate the meaning of fractions by students. The fractions were interpreted as part-whole and ratio but were difficult to perceive as numbers. Cramer et al. (2018) used experimental research to find out how elementary school students develop ideas about fractions using number lines. It was found that students are able to understand the number line as a model for fractions, especially for students with good concrete representation experience. In addition to the quantitative-based studies, Bayaga and Bossé (2018) had a case study combined with discourse analysis to investigate the performance of elementary and middle school students regarding fractions and their operations, as well as interpret students’ understanding from the point of view of the semantics and syntax of fractions. This study unraveled that semantic (conceptual) and syntactic (procedural) understanding of fractions are two things that must complement each other in learning. Kor, Teoh, Mohamed, & Singh (2019) applied a qualitative approach with clinical interviews to identify the definition of fractions by elementary school students. It was confirmed that students with moderate and low mathematics learning achievement tend to be weak in understanding concepts, visualization, strategies, and rationalization of fractions.

In contrast to the aforementioned studies, the current study employs phenomenological hermeneutics to identify the students’ developed meaning of fractions. The selection of this method is considered quite representative and distinctive compared to other methods since it focuses on how participants interpret the meaning and experience of meaning to a phenomenon (Ramezanzadeh, Adel, & Zareian, 2016; Stephenson, Giles, & Bissaker, 2018). In this case, the method is used to reveal how students construct the meaning of fractions and their experiences in doing so. In addition, preceding studies have elementary school students as participants, while this study investigated secondary school students. The two participants have different characteristics regarding experiences and knowledge, especially in learning fractions. The secondary school students have learned fractions in elementary school, so they already have a concept definition related to fractions (Tall & Vinner, 1981). For the differences, it is compelling to see whether secondary school students have the distinct or similar meaning of fractions as the primary students have (Martinez & Blanco, 2021). There are three questions that sought to be answered in this study: (RQ1) what is the meaning of fractions constructed by secondary school students? (RQ2) how do they use models to represent fractions? and (RQ3) how do they develop the meaning of fractions?

Theoretical Review

Interpretations and representations of fractions

Fractions have five interpretations; part of the whole, quotients, ratios, operators, and measures (Lamon, 2020). Fraction as part of a whole is interpreted as one or more parts of a whole. This interpretation tends to refer to proper fractions and is related to another meaning, namely fractions as a result of a division process (quotients). A part will never exist without going through the division process first. Fractions also can be expressed as the comparison of two objects or numbers (ratios). The object can be related or not. Moreover, it can be a measure or value of an object. For example, 50 cm can be written in the form 1/2 m. As an operator, fractions refer to an integer or line segment operated with a fraction, which will produce a new integer or line segment that is getting bigger or smaller. For example, $4 \times 15/2 = 30$ (4 turns into 20) or $15 \times 1/5 = 3$ (15 turns into 3). Fractions as part of a whole are more commonly found in mathematics textbooks and taught by teachers in the classroom (Getenet & Callingham, 2021; Martinez & Blanco, 2021).

Previous research revealed that elementary school students tend to interpret fractions as part-whole (Murdock-Stewart, 2005), which then becomes an obstacle for the students when interpreting improper fractions. They find it difficult to make a graphical representation of the fraction because the numerator is greater than the denominator (Singh et al., 2020). Basically, the limited meaning developed by students is because they have just learned about fractions. In fact, at the elementary school level in Indonesia, students are only introduced to fractions as part-whole (MoEC, 2018). In general, there are three models, also called external representation, used by teachers to represent fractions at schools; area model, number line, and sets of objects. Each model has characteristics directly associating with fraction meanings. Fractions as part-whole and quotient relate to the area model (Naik & Subramaniam, 2008; Tian, Bartek, Rahman, & Gunderson, 2021). The number line represents fractions as measures (Tian et al., 2021) and operators (Getenet & Callingham, 2021). Meanwhile, sets of objects can be used to explain fractions as a ratio (Van de Walle, Karp, & Bay-Williams, 2018). In Indonesia, secondary school is the last level of education for students to learn fractions. Therefore, students at that level should be able to interpret fractions, not only as part-whole but also include other meanings. This is because secondary students have better learning experiences and cognitive levels. To confirm and obtain empirical evidence on this matter, this study was carried out.

Hermeneutic phenomenology

Phenomenology is one type of research that seeks to explain the structure, meaning, and essence of life experienced by a person or group of people around a phenomenon (Breiger, 1995; Creswell & Creswell, 2018; Isnawan et al., 2022; Laverty, 2003). There are three types of phenomenology, one of which is hermeneutics (Palacios & Simons, 2021). Hermeneutics is a method to uncover hidden meanings. Therefore, hermeneutic phenomenology is not merely to understand a phenomenon as a whole but also to interprets the subjects' experiences (Desjarlais & Throop, 2011; Iared, Oliveira, & Payne, 2016; Keshavarz, 2020; Ramezanzadeh et al., 2016). In this case, the hermeneutic phenomenology in this study then refers to a method used to find out how students interpret a phenomenon (fraction) and how experiences lead the students to interpret the phenomenon.

The hermeneutics phenomenology is a fairly representative method in analyzing meaning, including the meaning of fractions, compared to other methods, especially in philosophical and methodological terms (Ramezanzadeh et al., 2016). In addition to directly providing information about how students interpret fractions, it is also able to determine whether the interpretation given by students is classified as an obstacle or not (Wyman, 2012). Besides that, hermeneutics phenomenology also tries to explore students' experiences that lead to the obstacles in interpreting fractions (Desjarlais & Throop, 2011). These results will then lead researchers to conclusions about the factors that cause students to experience obstacles. However, hermeneutic phenomenology also has some weaknesses, such as the number of participants tends to be small, the limited context that participants have in interpreting phenomena, the information provided tends to be hidden when there is no inter-subjective between the researchers and the participants, and unable to map out solutions in overcoming obstacles experienced by someone in interpreting a phenomenon (Standing, 2009; Stephenson et al., 2018).

Methods

Research design

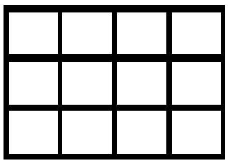
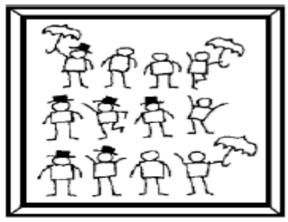
This study employed the hermeneutic phenomenology (Breiger, 1995; Tan, Wilson, & Olver, 2009) since it aims to interpret the meaning of fractions constructed by students and explores their experiences in obtaining that meaning. We acted as the main instrument in this study. Indeed, we are directly involved in all data collection processes (Creswell & Creswell, 2018). The supporting instruments include a fraction comprehension task, students' answer sheets, interview guidelines, and documentation (photos, audio recordings, and videos). The task aims to find out how students interpret fractions. From the task, we obtained students' answer sheets which would be analyzed using thematic analysis. The students' answers were then photographed for inclusion in the NVivo-12. To confirm the results of the students' answers, we conducted interviews and recorded the activities in order to obtain audio and video data. From the recordings, we made transcripts stored in a document format that would be inserted into NVivo-12. The results of audio and video recordings are also included in NVivo-12 to make it easier when there is insufficient interview transcript data.

Participants and data collection

We initially planned to have 35 eighth-grade students (13-15 years old) and a math teacher (28 years old with six years of teaching experience) who taught fractions to these students at the previous grade level as the participants. However, because the research was conducted during a pandemic and mathematics learning was conducted online, not all participants were willing to take the test. There were 29 students who took the test, and only 22 of them submitted the answers. For the interviews, at first, we planned to select three students who have different categories of mathematics abilities (low, medium, and high). This categorization is intended to examine if there are differences between categories regarding the interpretations of the fractions. We also divided the participants into three categories of mathematical ability based on report cards, namely low (mathematical score < 60), medium (mathematical score 60 – 79), and high (mathematical score 80). However, the interviews were administered, the data obtained from these sources was still lacking, so we decided to involve one more student in each category. In addition to the students' interviews, we interviewed the mathematics teacher because several students mentioned that the meaning of the fractions they had was developed through the teacher's instruction. Therefore, the number of participants in this study was six students and one teacher. Before administering the interviews with students, we explained the interview's purposes, asked permission from their parents, and had written expressions to not share the participants' identity as an important aspect of research ethics (Roberts & Allen, 2015).

Most of the data were collected online except for the teacher's interviews. The data collection began with collecting student biodata using google-forms to identify their general characteristics such as age, ethnicity, language, and mathematics abilities. After that, we gave the test to examine students' constructed meaning of fractions. An in-depth interview was then carried out after analyzing the students' answers to confirm what they did in the test (Deterding & Waters, 2018). Basically, such an interview is able to confirm students' answers and explore their understanding of fractions more deeply because of using semi-structured questions (Brown & Danaher, 2017). After analyzing the students' interviews, we interviewed the teacher to identify what factors make students have limited meanings of fractions.

Table 1. The task used to reveal the meanings of fractions

Task 1	Task 2	Task 3	Task 4
What is the meaning of a fraction? Provide an explanation!	Use the given models to show $\frac{1}{2}$ or $\frac{1}{4}$! 		

Mathematics tasks

The task used in this study consist of four items. The selection of the task is meant to support the emergent of various meaning of fractions. In addition, such a task tends to be more appropriate to the communication style and level of students' cognitive development (Chan & Clarke, 2017). Task 1 is used to reveal the meaning of fractions. We argue that the question in task 1 is appropriate to be asked to secondary school students since they have constructed or have a concept definition related to fractions. Task 2-4 aims to reveal the process of students interpreting fractions in three different representations. The three representations refer to different meanings of fractions; part-whole and quotient for the area model, measure and operator for the number line, and ratio for the sets of objects. In this case, students may reveal more than one fraction meaning. The first research question will be answered through task and task 2-4 is for the 2nd research question. The second question was used to validate the meaning of fractions revealed by the participants in task 1. There might be different results of the two questions, for example, a participant revealed that fractions are as ratio. However, she/he could not show the meaning using a set of objects. In this case, the meaning developed by the participant was not consistent. That is to say, there is a gap between the expressed meaning with her/his understanding of the fraction meaning. The third research question was answered based on the results of interviews with teachers and students. The tasks were referred to prior works

(Greenberg, 1996, p. 12; Iulia & Gugoiu, 2006; Petit, Laird, & Marsden, 2010). The details of the task can be seen in Table 1.

Data analysis

A thematic analysis with the aid of NVivo-12 was used to analyze the data. The software is considered very easy to use and accepts all types of data, making it easier to establish initial codes and themes. In addition, it is very helpful when reopening data to conduct audit trails, provide a variety of interesting output displays, and tend to be more in line with thematic analysis because it is systematic (Dalkin, Forster, Hodgson, Lhussier, & Car, 2020). The stages of thematic analysis in this study are as follows (Benavides-lahnstein & Ryder, 2019; Nowell, Norris, White, & Moules, 2017).

Familiarizing with data

At this first stage, we read the students' and interview transcripts repeatedly in order to be familiar with the data, so that the initial codes can be determined and predict the characteristics of the codes. In addition, we also repeatedly played and listened to the audio and video recordings to ensure that the interview transcripts were in accordance with the actual interview results. We have started using NVivo-12 because students' answers (in .JPG format) and interview transcripts (in .docx format) have been inputted into the software.

Generating initial codes

We identified the initial codes using NVivo-12, making nodes according to the number of variations of students' answers. For task 1, 6 nodes were obtained for students with the low category, 8 nodes for students with the moderate category, and 4 nodes for students with the high category. Some examples of the initial code for task 1 from low category students are "a fraction is a number consisting of a numerator and denominator" and "a number in the form of a per b" Meanwhile, for students with low, medium, and high categories respectively, the number of nodes is 3, 7, and 3 for task 2; 3, 7, and 3 for task 3; and 4, 6, and 4 for task 4. The initial code examples of students with the moderate category for task 2 are "shading but the number of areas is shaded incorrectly" and "delete several areas and shade one area." The sample initial code for high category students for task 3 is "labeling the number line with a wrong fraction" and for low category students in task 4 is "crossing out several people and the number crossed out does not match."

Searching for and reviewing themes

At this stage, the initial codes having the same characteristics are categorized into one node. These nodes will then become the theme. After the theme was formed, we also re-checked the contents of the theme nodes to ensure that there was no initial code that entered the inappropriate theme. For task 1, 2 theme nodes were made for students with the low category, 2 theme nodes for the moderate category, and 1 theme node for the high category. For tasks 2, 3, and 4, the number of theme nodes is 2 for all categories; 3 for all categories; and 2, 3, 3 for the high category. For example, the two initial codes of task 1 mentioned in the previous stage have the same characteristics, so the two initial codes belong to one theme. The themes have not been named.

Defining and naming themes

After the theme was formed, we defined and named the themes. One example of the themes for task 1 is "fraction as a ratio" which means a fraction as a comparison of two numbers or objects. The naming of the themes is based on our interpretation, which refers to theories regarding the meaning of fractions (Lamon, 2020). The example of a theme from tasks 2, 3, and 4 is "already able to make illustrations, but wrong," which refers to the initial codes.

Findings and Discussion

RQ1: What is the meaning of fractions developed by secondary school students?

Table 2 shows the initial codes generated from the students' answers and interview transcripts. In this study, references were used as an indicator to determine the number of reference sources that formed the initial codes. It refers to data derived from students' answers or interview transcripts. From the six initial codes, there are several codes that have the same characteristics and form a theme. There are two themes formed; T1 (IC-1L1, IC-1L2, IC-1L3, with IC-1L4) and T2 (IC-1L5 with IC-1L6). The characteristics of T1 is that fractions consist of two numbers separated by a "per," while T2 refers to the benefits of fractions, both for life, and for mathematics itself. These benefits are identical to fractions as a tool. Therefore, T1 and T2 are given the names of fractions as ratios and tool, respectively.

Table 2. The initial codes (IC) of task 1

Categories of students	IC	Description	References
Low mathematics ability (1L)	1	Numbers with a numerator and denominator	6
	2	Rational numbers	3
	3	Numbers in the form of **a** per **b**	3
	4	Writing a fraction and informing the numerator and denominator	2
	5	It is one topic in mathematics to divide	2
	6	It is used to divide anything	2
Mid-level mathematics ability (1M)	1	Numbers with a numerator and denominator	10
	2	Rational numbers	2
	3	Numbers in the form of **a** per **b**	5
	4	The quotient of whole numbers and positive integers	5
	5	Fractions as integers	2
	6	It is used to divide anything	3
	7	Divided each other	3
	8	The way to simplify the numerator and denominator	4
High mathematics ability (1H)	1	Numbers with a numerator and denominator	8
	2	Rational numbers	2
	3	Numbers in the form of **a** per **b**	9
	4	The results when dividing two numbers	2

The initial codes generated from students with the medium category are shown too in Table 2. Akin to students with the low category, the students tend to interpret fractions more as "numbers with a numerator and denominator." T1 is developed from IC-1M1, IC-1M2, and IC-

1M3 and T2 is from IC-1M6, IC-1M7, and IC-1M8. The IC-1M4 forms its own theme (T3) while KA-1M5 is named as T4. T1 and T2 have the same characteristics as the low student category, so the definition and name of the theme is the same as that of the category. T3 refers to the meaning of a fraction as a quotient. T4 refers to the meaning of fractions as integers. Table 2 also shows the generating codes of the students in the high category. The students tend to interpret fractions as "numbers in the form of a per b" rather than "numbers with a numerator and denominator." Two themes were formed; T1 represents IC-1H1, IC-1H2, and IC-1H3 and T3 refers to IC-1H4. T3 in this case has the same definition and name as the category of mid-level students- fraction as a quotient. Table 3 summarizes the themes about the meaning of fractions developed by different categories of secondary students.

Table 3. The generated themes of the meaning of fractions

Codes	Themes	Low	Medium	High	Notes
T1	Fraction as ratio	√	√	√	Correct
T2	Fraction as a tool	√	√	-	Incorrect
T3	Fraction as quotients	-	√	√	Correct
T4	Fraction as integers	-	√	-	Incorrect

RQ2: How do secondary school students use models to represent fractions?

Table 4. The initial codes (IC) of task 2

Categories of students	IC	Description	References
Low mathematics ability (2L)	1	Make a shade but the number of the shaded area is incorrect	7
	2	Write other fractions	4
	3	No answers	2
Mid-level mathematics ability (2M)	1	Make a shade but the number of the shaded area is incorrect	4
	2	Delete some areas and shade one area	3
	3	Write other fractions	9
	4	Write an equivalent fraction	3
	5	Re-write the task and write a fraction that equals to 1	2
	6	Explain the areas that should be shaded correctly	1
	7	Shade the areas correctly	1
High mathematics ability (2H)	1	No answers	2
	2	Write other fractions	2
	3	Shade the areas correctly	5

Task 2 (area model)

Table 4 shows several initial codes for students with low mathematics ability. In this case, two themes are generated. T21 comes from IC-2L1, which refers to students who make illustrations, but the illustrations are incorrect. T22 represents that students who cannot make illustrations. They do not try to shade or color the area model, but write down other fractions and leave the answers blank. Some initial codes for students with mid-level mathematics ability shown too in Table 4 were formed and converted into three themes. T21 is from IC-2M1 and

IC-2M2, while T22 represents IC-2M3, IC-2M4, and IC-2M5. There is a new theme (T23 comprises IC-2M6 and IC-2M7) named “make illustrations correctly.” However, this case differs slightly from the previous case in terms of the number of references. The references on T22 are more than T21. In other words, there were more mid-level students who could not make illustrations than students who were able to make incorrect illustrations.

The number of initial codes for students with high mathematics ability formed is shown also in Table 4. Two themes were formed, namely T22 (students are not able to make illustrations) and T23 (students were able to make illustrations correctly). T22 consists of IC-2H1 and IC-2H2, while T23 only consists of IC-2H3. The references are dominated by T23, so it can be concluded that there are more students who are able to make illustrations correctly than students who cannot make illustrations. Table 5 highlights the generating themes on how students construct the meaning of fractions using area model. In this case, most of the students cannot make illustrations.

Table 5. The generating themes of the task 2

Codes	Themes	Low	Medium	High
T21	Make incorrect illustrations	7	7	0
T22	Cannot make illustrations	6	14	4
T23	Make correct illustrations	0	2	5

Task 3 (number lines)

Table 6. The initial codes (IC) of task 3

Categories of students	IC	Description	References
Low mathematics ability (3L)	1	Label the number line with integers	4
	2	Label the number line with incorrect fractions	2
	3	Write other fractions	7
Mid-level mathematics ability (3M)	1	Label the number line with incorrect fractions	4
	2	No answers	6
	3	Write other fractions	9
	4	Write decimals	1
	5	Write an equivalent fraction	3
	6	Re-write the task and write a fraction that equals to 1	2
	7	Shade half of the number line	1
High mathematics ability (3H)	1	Label the number line with incorrect fractions	2
	2	No answers	2
	3	Label the number line correctly	5

There are 3 initial codes for students with low mathematics ability (Table 6). IC-3L1 and IC-3L2 have the same characteristics; students make incorrect illustrations. Therefore, the two initial codes belong to one theme (T31). Meanwhile, IC-3L3 refers to T32; students cannot make illustration. In this case, there are more T32 references than T31, meaning that most students tend not to be able to make illustrations using number lines. Table 6 also shows the initial codes for students with mid-level mathematics ability.

The IC-3M1 refers to T31, while IC-3M2 to IC-3M6 has the same characteristics with T32. IC-3M7 represents a new theme (T33); students are able to make illustrations correctly. The results are almost the same as the case of students with low category. There are 3 initial codes formed for students with high mathematics ability in Table 6 too. Three themes were formed because each initial code had different characteristics (T31 with IC-3H1, T32 with IC-3H2, and T33 with KA-3H3). T33 represents the students who are able to make illustrations using number lines. Table 7 sums up the themes on how students use number lines to develop the meaning of fractions. In case of number lines, only few students make correct illustrations.

Table 7. The generating themes of the task 3

Codes	Themes	Low	Medium	High
T31	Make incorrect illustrations	6	4	2
T32	Do not make illustrations	7	21	2
T33	Make correct illustrations	0	1	5

Task 4 (sets of objects)

Table 8 shows the initial codes for students with low mathematics ability. IC-4L2 to IC-4L4 has the same characteristics, namely students cannot make illustrations (T42) while IC-4LI refers to T41 where students make incorrect illustrations. Overall, most of the students are not able to make correct illustrations. There are six initial codes for students with mid-level mathematics ability (Table 8).

Table 8. The initial codes (IC) of task 4

Categories of students	IC	Description	References
Low mathematics ability (4L)	1	Cross out incorrect numbers of people	2
	2	Register and move objects	2
	3	Write other fractions	4
	4	No answers	4
Mid-level mathematics ability (4M)	1	Draw some people	2
	2	Write the required fractions	2
	3	Write other fractions	9
	4	No answers	6
	5	Write an equivalent fraction	3
	6	Explain the number of objects correctly	1
High mathematics ability (4H)	1	Cross out incorrect numbers of people	2
	2	Take one object as numerator and another as denominator	2
	3	Write two equivalent fractions	2
	4	Mark correct numbers of objects	4

IC-4M1 refers to T41, that is, students make incorrect illustrations. IC-4M2 to IC-4M5 refers to T42, which represents students cannot make illustrations. IC-4M6 leads to T43 which means students are able to make illustrations correctly. Table 8 also provides information about the number of initial codes. There are 3 themes; T41, T42, and T43. T41 consists of IC-4H1 and IC-4H2, T42 only consists of IC-4H3, and T43 also only consists of IC-4H4. In short, the number of students who are able to make incorrect illustrations is the same as the number of students who are able to make illustrations. Table 9 includes the generating themes about how students construct the meaning of fractions using sets of objects. As in task 2 and task 3, for task 4, most students cannot make illustrations.

Table 9. The generating themes of the task 4

Codes	Themes	Low	Medium	High
T41	Make incorrect illustrations	2	2	4
T42	Do not make illustrations	10	20	2
T43	Make correct illustrations	0	1	4

The differences in developing meaning of the fractions

Referring to the third theme (T23/T33/T43) and linking the theme to the espoused meanings of fractions, we compare (Table 10) what the students espoused and their use of models to represent the fraction. It aims to examine the consistency between the expressed meaning of fractions and their representations using models. In short, the table reveals students' inconsistency between what they espoused and the use of models to represent the meaning of fractions.

Table 10. The comparison between the students' espoused meaning of fractions and the use of models to represent the meaning

Categories of students	The meaning of fractions (RQ1)	The use of models to represent fractions		
		Area model	*Number line*	*Sets of objects*
Low mathematics ability	Most of the students espoused ratio	All of the students cannot represent the meaning using the model	All of the students cannot represent the meaning using the model	All of the students cannot represent the meaning using the model
Mid-level mathematics ability	Most of the students espoused ratio and quotient	The students mostly cannot represent the meaning using the model	The students mostly cannot represent the meaning using the model	The students mostly cannot represent the meaning using the model
High mathematics ability	Most of the students espoused ratio and quotient	The students mostly can represent the meaning using the model	The students mostly can represent the meaning using the model	The students mostly cannot represent the meaning using the model

RQ3: How do secondary school students develop the meaning of fractions?

The findings of the first two questions show that secondary school students have false interpretations, limitations and inconsistencies in interpreting the fractions. In this case, the students have obstacles in developing the meaning of fractions. To further understand this case, we had in-depth interviews with the students. Transcript 1 unravels that the first students with low mathematics ability do not know the meaning of fractions, the task, and how the teacher teaches fractions. Since the obtained information from LI is limited, we interviewed other students (L2). Transcript 1 also provides quite a lot of information, such as students know that fractions have the meaning as rational numbers. They know it from textbooks. For task 2, students can make illustrations, while for tasks 3 and 4, students cannot accomplish them because of incomprehension of the tasks. They admitted that in learning fractions, the teacher had not introduced number lines and a collection of objects and tended to teach directly by giving formulas, then providing examples and exercises.

Transcript 1. The excerpts of interview with students in low category

Students	Questions	Answer
L1	*What is a fraction?*	*Fraction? ...*
	Do you understand task 2 and 3?	*...*
	Have you ever had these tasks before?	*Yes*
	What tool did your teacher use to teach fractions?	*Cake*
	How did the teacher use the cake?	*I have no idea*
L2	*What is a fraction?*	*A rational number is written in a per b*
	How do you know?	*From, ..., my notes.*
	How did your teacher teach fractions?	*Firstly explain, if we have understood what he had explained, then we were given worked examples of word problems and eventually some exercises.*
	How do you shade task 2?	*I shaded 4*
	Why did not solve task 3?	*I do not understand*
	Has your teacher given tasks like task 3 and task 4?	*Never, we ever had task 2*

Transcript 2 shows that the meaning of fractions that first students (M1) with mid-level mathematics ability had come from notebooks. In other words, the meaning was perceived from their teacher. Other important points are they could not make the illustration, and their teacher commonly used number lines. To strengthen trustworthiness, we interviewed other students (M2) in order to obtain the same or different justification and from different sources. Transcript 2 also unravels that the teacher teaches directly and has used models, such as in task 2-4. However, of the three models, the area model is most often used by teachers when teaching fractions.

Transcript 2. The excerpts of interview with students in mid-level category

Students	Questions	Answer
M1	*What is a fraction?*	*Can divide each other by simplifying the numerator and denominator*
	Is it from your notes?	*Yes, my notes from the previous semester*
	You are required to make illustrations for task 2, how if it is 1/4?	*It was like a task in primary school, I know 1/4 but task 3 ….*
	How about task 3?	*First of all, it is 1/4 then change it, 1/0, 1/4, 1/2, 3/4 and finally 1.*
	Task 4?	*I just look for the same pictures*
	Which model did you ever use?	*Task 3*
	The number line?	*Yes, but I do not understand*
M2	*How did your teacher teach fractions?*	*Hmm, we were given some explanations then exercises*

	Then you were given some explanations then worked examples?	*Yes*
	Has your teacher given models like task 2, 3, and 4?	*Yes*
	Which one is the most used?	*Area model*

Transcript 3 shows that the meaning of fractions for first students (H1) with high mathematics ability is constructed through what the teacher has taught. In addition, the teacher never used illustration, and the learning methods used tended to be direct learning. Transcript 3 is different from those informed by students in the low and medium categories. In this case, we conducted an interview with another student who was in the high category (H2). Transcript 3 also indicates that the meaning of the fractions owned by students came from the teacher's explanation and searching on the internet. The students have been given a number line model, but rarely did the teacher provided a set of objects model.

Transcript 3. The excerpts of interview with students in high category

Students	Questions	Answer
H1	*What is a fraction?*	*Fraction is a number that has a numerator and denominator*
	Is there any other meaning of fractions?	*…(Shaking heads)*
	How do you know the meaning of the fraction?	*My teacher*
	Has your teacher ever defined a fraction?	*Yes*
	Has your teacher asked you to make illustrations for a fraction?	*Never*
	How did your teacher teach fractions?	*The teacher firstly explained a worked example and then followed by some exercises.*
H2	*What is a fraction?*	*It is a number written in a per b, where a and b are integers, b does not equal to 0 and a is not the same as b*
	How do you know?	*From the teacher's explanation and internet*
	For task 2, how many do you shade for ½?	*6.*
	Has your teacher ever introduced number lines for fractions? A set of objects?	*Yes, but only on few occasions. Never.*

In general, the results of interviews with all categories of students provide information that the meaning of fractions that students have comes from their teacher. In addition, it is found that the teacher mostly used the area model when making illustrations, while the number line was rarely used, and the set of objects was never used. To ensure this, we interviewed the teacher (Transcript 4). It shows that the teacher has incorrect interpretations of a fraction. This is what may cause students to experience obstacles in interpreting fractions. In addition, the teacher also revealed that he used the area model quite often. The number line model was once used by the teacher, but it was discontinued due to the difficulty faced by students in using and understanding the number line.

Transcript 4. The excerpts of the interview with the teacher

Researcher(s)	Teacher
What is a fraction?	*The fraction is …let me think first. The fraction is like this, in daily life, to help us to divide something*
Another meaning?	*It is like that, it has many*
How did you teach fractions?	*A direct learning and group discussion*
What did you use to make illustrations for fractions?	*Sometimes I brought … a box, directly using the whiteboard, but most of my students understood.*
Have you ever used a number line?	*Yes, I have, but they were confused. It was a bit difficult to use the number line. So, I used another model.*
Area mode?	*Yes*
Did you use the area model for all fractions topics?	*No*

Basically, secondary school students are able to interpret fractions, although they do not refer to all the meanings of fractions. Some students with low and mid-level mathematics ability interpreted fractions as a tool and integers. These interpretations are certainly not correct. The majority of the students with all categories tend to have a ratio as the meaning of fractions following by quotient. However, the expressed meaning responding to task 1 is not supported by their ability to use the models to represent the meaning. For instance, although the students in all categories were able to interpret a fraction as a ratio, most of them could not represent the fraction using the related model (sets of objects). This study found that only some students with high mathematics ability, who interpret fractions as quotient, are able to correctly use the area model to represent the fraction. Therefore, there is an inconsistency between the students' espoused meaning and their use of models to represent the meaning.

Because most of the students in this study are not consistent, it can be concluded that they experienced difficulties in interpreting fractions. The results of interviews with students show that these obstacles occurred since they did not understand the tasks since such tasks were rarely used in fractions instructions. Moreover, they had false interpretations of fractions from their teacher. These are confirmed with the results of the teacher's interviews (Transcript 4). The teacher understood fractions as a tool that can make it easier for someone to divide something. In fact, none of the interpretations leads to part-whole, size, ratio, operator, and quotient. In other words, the mathematics teachers in this study have not been able to reach the stage of scholarly knowledge (Frejd & Bergsten, 2018). The students' obstacles in interpreting fractions may be caused by the mathematics teacher. In this case, the teacher does not understand the meaning of fractions properly, which affects the students' understanding. This type of learning barrier is classified as a didactical obstacle because it relates to content knowledge (CK) or the teacher's erroneous conceptual understanding (Brousseau, 2002). The teacher also rarely uses models in teaching fractions, so students tend to experience obstacles when making illustrations. Besides that, the teacher frequently uses conventional methods in learning, which gives rise to an epistemological obstacle relating to pedagogical content knowledge (PCK) in terms of limited context or models that teachers use in learning (Suryadi, 2019). Prior studies revealed that when teachers have inadequate CK and PCK regarding fractions, it will have an unfavorable effect on students' understanding of fractions or otherwise (Kazemi & Rafiepour, 2018; Veloo & Puteh, 2017; Wahyu, 2021). Copur-Gencturk (2021) also found that teachers lack an understanding of fractions.

Fractions as part-whole do not appear in this study. This is different from the meaning of fractions that elementary school students develop; fractions as part-whole, and this meaning is taught using the area model (Gabriel et al., 2013; Martinez & Blanco, 2021). Fractions as ratios and quotients are more abstract (formal) than the part-whole. The students' cognitive development at the secondary school level is also in the formal operational stage (Babakr, Mohamedamin, & Kakamad, 2019). This is what might cause students in this study to interpret fractions differently.

When compared to students in other categories, students in the low category in this study had a fairly limited meaning of fractions. They are only able to interpret fractions as ratios, while students in other categories are able to interpret fractions as ratios and quotients. This result may be due to the fact that most of the students in the low category do not remember the teacher's explanation and the illustrations that the teacher has used in learning. Evidently, based on Table 10, none of the students in the low category were able to illustrate 1/2 or 1/4 correctly.

The students do not interpret fractions as a measure in this study. This finding is due to the fact that teachers rarely use number lines in learning fractions. In Transcript 4, the teacher revealed that he had used the number line model, but students tended to have difficulty and confusion when given the model, so they never used it again. The number line is not a new model in learning fractions because the model has been introduced in elementary schools (MoEC, 2018). If it is associated with the stages of cognitive development of secondary school students, then the number line seems quite appropriate to be used as an illustration in learning fractions. The number line can also cover the weakness of the area model, which tends to refer to the proper fraction (a fraction that is less than 1). This model can easily represent various forms of fractions, both proper and improper fractions, because it can be extended (Iulia & Gugoiu, 2006). Fractions as operators do not appear either in this study. This meaning is quite abstract because it relates to the stretching and shrinking of numbers (Getenet & Callingham, 2021). In addition, it also involves multiplication operations, so it becomes more complicated. Actually, fractions as operators can still be learned by students using the number line (Isnawan, Suryadi & Turmudi, 2022) when learning about the multiplication of fractions.

Conclusion

This study found that most of the secondary school students in various categories of mathematics ability tend to interpret fractions as ratios and quotients, albeit inconsistency between the espoused meaning and the use of models is found. The interpreted meanings are different from what primary students have. The limitation and inconsistency of the secondary students' meaning of fractions may be due to the teacher's inadequate content knowledge on fractions. Moreover, the teacher interprets fractions as a tool to divide something. This condition is referred to as a didactical obstacle (Marfuah et al., 2022). Another factor is less attention on the use of models in fraction instructions. Indeed, it hinders students' chance to have various meanings of fractions. The use of multiple models should be encouraged to support students' understanding of the meaning of the fraction. By way of example, number lines are utilized to develop the meaning of fractions as measures and operators or sets of objects for ratio.

In addition to the aforementioned factors, mathematics textbooks might also affect the students' limited interpretations. For instance, one of the electronic mathematics textbooks used in secondary schools (As'ari, Tohir, Valentino, Imron, & Taufiq, 2017) ignores the importance of developing the various meaning of fractions. In fact, the textbook already uses area models and number lines as illustrations in solving problems regarding fractions. In other words, the textbook has not explored didactical uses of models. However, this will be a chance for further research. Even at the university level, the meaning of fractions does not seem to get much attention. For example, the number theory course does not discuss fractions and their meanings but discusses rational numbers (Rosen, 2011). The findings of this study imply that (mathematics) teacher education should also focus on developing the prospective teachers' various meanings of fractions.

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