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Research articles

Exploring students' algebraic thinking in generational activities and their difficulties

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Abstrak Kegiatan generasional, transformasional, dan meta-level global adalah tiga kegiatan berpikir aljabar yang dilakukan siswa di sekolah. Beberapa penelitian sebelumnya menunjukkan bahwa siswa lebih banyak terlibat dalam kegiatan generasional dibandingkan kegiatan berpikir aljabar lainnya. Namun, terdapat beberapa siswa yang mengalami kesulitan dalam menyelesaikan masalah pada kegiatan generasional. Oleh karena itu, penelitian ini akan difokuskan pada kegiatan generasional, dengan tujuan menganalisis pemikiran aljabar siswa dalam aktivitas generasional dan kesulitan yang dialami siswa. Penelitian ini melibatkan 95 siswa kelas 7 yang mengikuti tes awal untuk mengukur kemampuan dalam kegiatan generasional. Jawaban siswa dan hasil wawancara dianalisis melalui tiga tahap, yaitu kondensasi data, penyajian data, serta penarikan dan verifikasi kesimpulan. Hasil penelitian ini menunjukkan bahwa dalam kegiatan generasional, siswa dapat mengeneralisasi pernyataan dari pola lebih baik daripada membuat pernyataan dan persamaan yang mengandung variabel. Siswa menunjukkan berbagai kesulitan dalam kegiatan generasional, diantaranya (1) memahami masalah dan mengubahnya menjadi bentuk matematis dan (2) menggeneralisasikan pola suku ke-n. Hal ini disebabkan siswa tidak memahami hubungan beberapa kondisi dalam masalah yang diberikan dan arti variabel.

Kata kunci *Berpikir aljabar, Kegiatan generasional, Kesulitan siswa*

Abstract Generational, transformational, and global meta-level are three typical activities of algebraic thinking students engage in school. Several studies show that students involve more in the generational activity than the other activities. However, some students still have difficulties solving problems in the generational activity. Therefore, this study focused on the generational activity, aiming to analyze student's algebraic thinking and difficulties in the generational activity. It involved ninety-five 7^{th}-grade students given an initial test to measure their abilities in the generational activity. The analysis of students' answers and interviews follow three steps; data condensation, data display, and drawing and verifying conclusions. This study indicates that, in the generational activity, the students can generalize statements from patterns better than forming statements and equations containing an unknown quantity. Students' difficulties in the generational activities, including (1) understand the problems and turn them into mathematical forms and (2) generalize the patterns to the n^{th} term. These are due to the students' incomprehension of the relationships of some conditions in the given problems and understanding of the meaning of variables.

Keywords *Algebraic thinking, Generational activity, Students' difficulties*

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Introduction

Students in 7th grade, especially in Indonesia, have been formally introduced to algebra. Algebra is an essential element of algebraic thinking (Freudenthal, 1977). However, algebra and algebraic thinking are two different things. Algebra is a product, whereas algebraic thinking is a process (Lins, 1992). Algebraic thinking links various mathematical topics to better understand through generalizing patterns, presenting relationships, and analyzing visible changes (Booker, 2009). There are three activities of algebraic thinking based on students' activities while solving algebra problems: generational, transformational, and global meta-level activities (Kieran, 2004). Agoestanto et al. (2019) found that the ability in generational activity supports students' global meta-level ability. It also has many meanings in the process of constructing algebraic objects (Kieran, 2004). It shows that generational activity plays an essential role in algebraic thinking.

Several studies (e.g., Permatasari & Harta, 2018) reveal that students have higher generational activity than the other activities, but some students undergo difficulties in the generational activity. Moreover, students' abilities are still insufficient when dealing with generational activities (Andini & Suryadi, 2017; Fakhrunisa & Hasanah, 2020; Muthmainnah, Priatna & Priatna, 2017). Some studies show different results where grade 7 can generalize patterns and use symbols (Britt & Irwin, 2008; Patton & Santos, 2012). Besides the importance of the generational activity, most of the algebraic thinking is the generational activity.

Considering students' deficient abilities in generational activities, the importance of the activities, along the students' difficulties, which can inhibit them in expanding their thinking to solve mathematical problems, the generational activities in algebraic thinking must receive special attention. Thus, this research is only focused on generational activity. Meanwhile, to further understand the students' difficulties, it is necessary to analyze the position and causes of the difficulties. Understanding the difficulties can improve students' algebraic thinking skills, especially in generational activities where these activities are pivotal and support algebraic thinking as an important part of mathematical thinking and mathematical reasoning. The present study aims to analyze students' ability and their difficulties in the generational activity of algebraic thinking.

Generational activities in algebraic thinking

Algebraic thinking relates to various cognitive strategies that can assist in mastering complex mathematical concepts (Windsor, 2010). It can be defined by emphasizing algebra as a generalization of arithmetic or defined in terms of functions (Groth, 2013). The ability to represent quantitative situations so that relations among variables become apparent is also called algebraic thinking (Panasuk & Beyranevand, 2010). Thus, algebraic thinking is the use of multiple representations to present, generalize and solve quantitative situations. According to Windsor (2009), it is a fundamental component in mathematical reasoning and thinking. There are three algebraic thinking activities based on students' activities while solving algebraic problems; generational, transformational, and global meta-level (Kieran, 2004). The generational activity includes expressions of generality arising from geometric patterns or numerical sequences or numerical relationships and equations containing an unknown to represent the situation of the problems (Kieran, 2004).

Forming statements and equations that contain an unknown quantity is the first aspect of generational activity. Radford (2011) describes algebraic thinking as dealing with indeterminate

quantities conceived of in analytic ways. One of the crucial aspects of algebraic thinking is interpreting and representing quantitative situations (Kieran, 1996; Lepak, Wernet, & Ayieko, 2018). To assess students' ability to form the statements and equations that contain an unknown quantity, this study used some problems where students need to generate equations to represent the quantitative relationships involved. The students' ability in generational activity is characterized by using symbols that are representations to resolve quantitative situations relationally by using symbols (Andriani, 2015). Quantitative situations usually involve additive propositions, relational propositions, or both. For example, "Jake and Tom have 30 marbles altogether" is an additive proposition. "One pound of shrimp costs $3.50 more than one pound of fish" is a relational proposition (Cai et al., 2011). To establish statements and equations, students must think relationally or use relation thinking when examining two or more mathematical ideas or objects, alternatively looking for connections between them (Molina, Castro, & Ambrose, 2006). Relational thinking is when students can build relationships of various objects/contexts related to each other (Steinweg, Akinwunmi, & Lenz, 2018). It is a thinking that utilizes the relationship between the elements in the sentence and the relationship of the arithmetic structure (Molina, Castro & Mason, 2008).

The second indicator of the activity is *generality arising from geometric patterns or numerical sequences, or numerical relationships*. The expression of algebraic objects is the patterns like numerical sequence patterns, geometric patterns, and formulas related to numerical solutions (Badawi, 2015; Suwanto et al., 2017). Pattern generalization is a core area in mathematics characterized by more strategic and reasoning knowledge than mathematical content knowledge (El Mouhayar & Jurdak, 2015, 2016). Lannin, Barker, and Townsend (2006) suggest that pattern generalization strategies often emerge from various ways of reasoning. For example, to extend the pattern, an additive strategy may emerge from two different ways of reasoning: (1) noticing that the number of squares increases by two each time: 3, 5, 7, 9, 11, 13, 15, 17, 19; and (2) recognizing the structural growth of the pattern and that two squares increase in the top and bottom rows by each step. Some previous studies (El Mouhayar & Jurdak, 2015, 2016; Radford, 2003, 2008; Rivera, 2010) demonstrate that students use different strategies and ways of reasoning to generalize patterns. Students' ability to generalize geometric patterns, numerical sequences, or numerical relationships can be accessed by utilized some problems where students need to generalize geometric patterns and represent these generalizations in various forms in symbolic expressions.

Students' difficulties in algebraic thinking

Teachers consider that algebra is one of the most critical areas of mathematics. Despite the importance placed on algebra in mathematics curricula, many students find it abstract and difficult to comprehend (Witzel, Mercer & Miller, 2003). The term difficulties in this study refer to obstacles that cause errors or mistakes made by students when dealing with algebra problems (Jupri & Drijvers, 2016). Students face learning challenges when embarking on algebra that forms a common set of basic understandings necessary to negotiate through multiple topics with varying sources of difficulty (Rakes et al., 2010). According to Kieran (2003), students' learning difficulties centered on the meaning of letters, the change from arithmetic to algebraic conventions, and the recognition and use of structures. Some relevant studies (e.g., Jupri, Drijvers & van den Heuvel-Panhuizen, 2014; Malihatuddarojah & Prahmana, 2019) show that students in Indonesia made mistakes in solving problems about algebraic operations such as in

variables, negative signs, solving algebraic equations, algebraic operations, and solving fractions. Moreover, students' deficiencies in algebraic thinking are on (1) interpreting information from the word problem to mathematical language, (2) comprehending the given information and the question in the problem, (3) combination of logic and concept they have learned when solving the problem (Muthmainnah et al., 2017)

Some studies (Cai et al., 2011; Walkington, Sherman & Petrosino, 2012; Fakhrunisa & Hasanah, 2020; Cahyaningtyas, Novita & Toto, 2018; Andini & Suryadi, 2017) use generational activities problems to identify students' difficulties in algebraic thinking. By analyzing students' answers when they solve it, they found that students have difficulty in interpreting and representing quantitative relationships, especially those involving relational proportion (Cai et al., 2011; Walkington et al., 2012; Fakhrunisa & Hasanah, 2020), obtaining information from the questions given, so students had difficulty predicting patterns and information chunking (Cahyaningtyas et al., 2018; Andini & Suryadi, 2017), and students are not accustomed to seeing the rules that exist in generalizing the pattern (Andini & Suryadi, 2017). In this study, the students' difficulties were identified through their answers to the given tasks and interviews, referring to the interpretation and representation of quantitative relationships and the prediction of patterns.

Methods

The current research follows a qualitative approach (Miles & Huberman, 1994) involving ninety-five 7th-grade students from three schools. The students were given a test to determine students' ability in generational activity. The test consists of two aspects of generational activities with three questions in each aspect (Kieran, 2004). Table 1 shows the indicators of the generational activity along with the tasks. We developed the test, and it has been declared valid by experts' assessment and has a reliability index of 0.677 (reliable). The results of students' tests were categorized into eight themes, for example, incorrect answers and no efforts in using symbols. Afterward, six students who represent each theme except for no answers were purposively selected to be interviews in a semi-structured way. The interview aims to confirm students' answers and reveal some points that students did not work in the problems, such as students' basic algebra
concepts.

Three stages of qualitative data analysis (Miles & Huberman, 1994) were applied in data analysis. In the first stage, data condensation, the students' answers for number 1-3 were coded into four themes referring to the indicators of the generational activity; correct answers (FSE-CA), wrong answers where there are efforts to use symbols (FSE-US), wrong answers and no effort in using symbols (FSE-NUS), and no answers at all (FSE-NA). Whereas, the student's answers for numbers 4-6 were also categorized into four: correct answers solved by using algebra (GP-CA); correct answers that are not solved by using algebra, calculations are used to generate answers, with or without explanation, or by using images to reinforce the calculations used (GP-CANUA); wrong answers (GP-WA); and no answers at all (GP-NA). The selected students' transcripts were coded inductively to identify their difficulties and sources of the difficulties. In data display, we used a method of describing participants through a role-ordered matrix to present the students' answers and interviews. In drawing the conclusion, we noticed patterns from the students' answers and making contrasts/comparisons from that with the interviews and

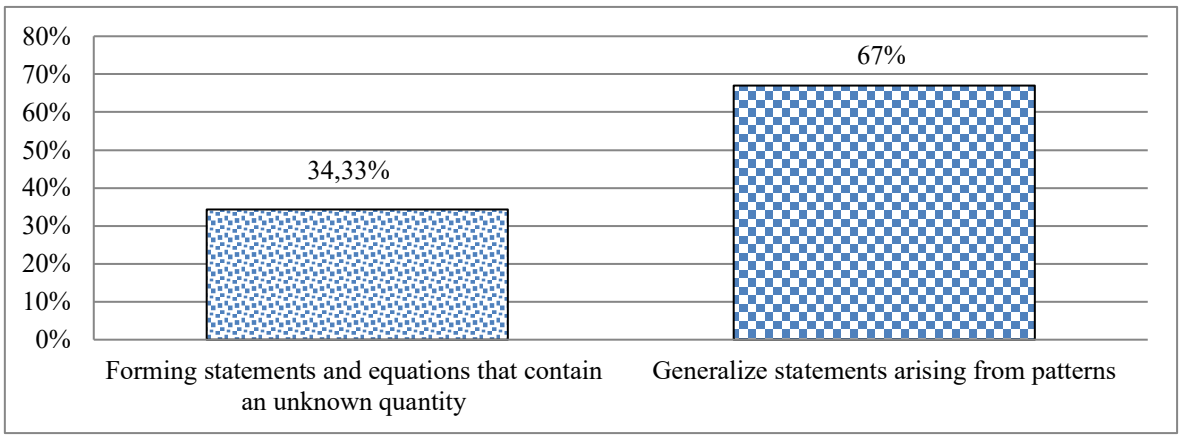
relevant studies. After obtaining a conclusion, a verification is carried out by employing a theoretical triangulation, which is considered appropriate for this research.

Table 1. The generational activity's indicators and tasks

Indicators	Tasks
Forming statements and equations that contain an unknown quantity	Consider the statement below! 1. Eko's pocket money is twice as much as Dwi's pocket money. If Eko's pocket money is m, write down Dwi's pocket money! 2. The temperature in Pontianak City is 10° higher than the temperature in Yogyakarta. The temperature in Pontianak is h. Write down the temperature in Pontianak! 3. Pak Slamet has one meter of cloth. For certain purposes, cut y cm. Write down the remaining cloth that Mr. Slamet has!
Generalizing statements arising from patterns	Consider the arrangement of shapes below!  (1) (2) (3) (4) 4. How many triangles are needed to make the 10^{th} shape? 5. What arrangement of shapes requires 30 triangles? 6. How many triangles are needed to construct the nth shape?

Findings and Discussion

Overall, the results of students' correct answers on the test are presented in Graph 1. It shows that students can generalize the statements from patterns better than forming statements and equations.



Graph 1. The summary of students' answers on the given tasks

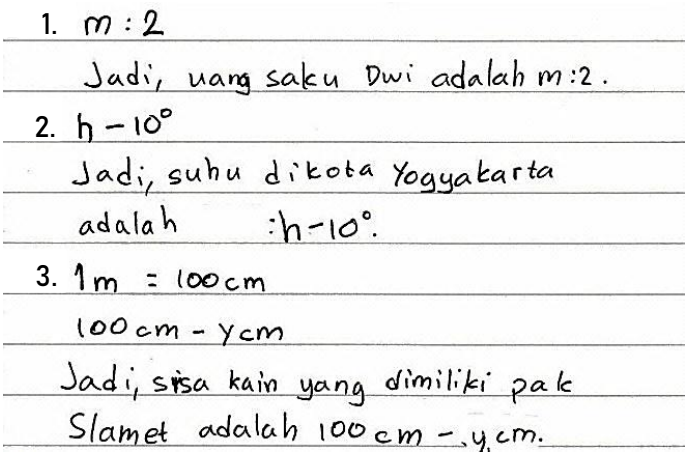
Forming statements and equations that contain an unknown quantity

Problem 1-3 aims to form statements and equations, including forming an equation containing an unknown quantity that represents a problem situation. The summary of students' answers is presented in Table 2. It shows that 34.33% of students had correct answers, meaning

that around one-third of the students can form statements and equations, which contain an unknown quantity or variable to represent the problem situation (Figure 1).

Table 2. Students' answers on problems 1 to 3

Code	Problem 1 (%)	Problem 2 (%)	Problem 3 (%)	Average (%)
FSE-CA	28	37	38	34.33
FSE-US	40	31	23	31
FSE-NUS	26	26	32	28
FSE-NA	6	6	7	6,33



Translation:

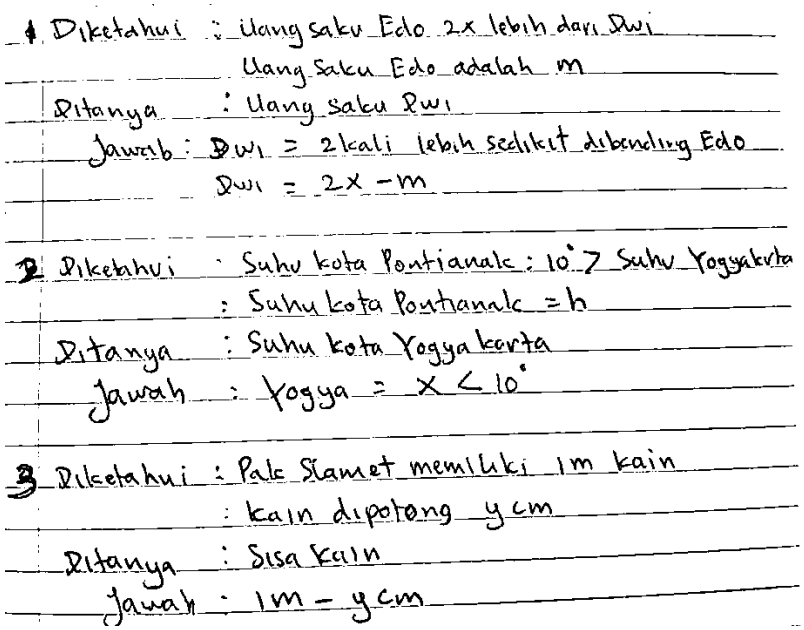
1. *m:2*
   Thus, Dwi's pocket money is m:2
2. $h - 10°$
   Thus, the temperature in Yogyakarta city is $h - 10°$
3. *1m=100 cm*
   100cm-y cm
   Thus, the remaining cloth that Mr. Slamet has is Answer: 100 cm-y cm

Figure 1. A sample of FSE-CA students' answers

On the other hand, 31% of the students had wrong answers, but there are efforts to use symbols (Figure 2). In problem 1, the FSE-US students understand that Eko's pocket money is twice as much as Dwi's pocket money but cannot write down Dwi's pocket money and ultimately gets the wrong answer. Similarly, in Problem 2, the temperature in Pontianak 10° is greater than the temperature in Yogyakarta City, but they could not write down the temperature of Yogyakarta city. This shows that students understand the given situation. In contrast to these two problems, problem 3 does not require reverse thinking skills.

Figure 2 shows that FSE-US student has not been able to interpret and represent quantitative relationships, especially those involving relational proportion. The students do not understand the meaning of more than and twice more. One aspect of algebra in which students can build relationships of various objects/contexts related to each other is relational thinking (Steinweg et al., 2018). Relational thinking utilizes the relationship between the elements in the sentence and the relationship of the arithmetic structure (Molina et al., 2008). Relational thinking describes an important part of algebra thinking. Many students can use algebra but do not understand the relationships of the given problem. Thus, students have difficulty understanding the problem and

turning it into a mathematical form. Cai et al. (2011) reinforced the statement that students have difficulty interpreting and representing quantitative relationships, especially those involving relational proportion. Walkington et al. (2012) found that students were largely unsuccessful in solving algebra word problems when they jumped directly from the problem text to representation without making sense of quantities and their relationships.



Translation:

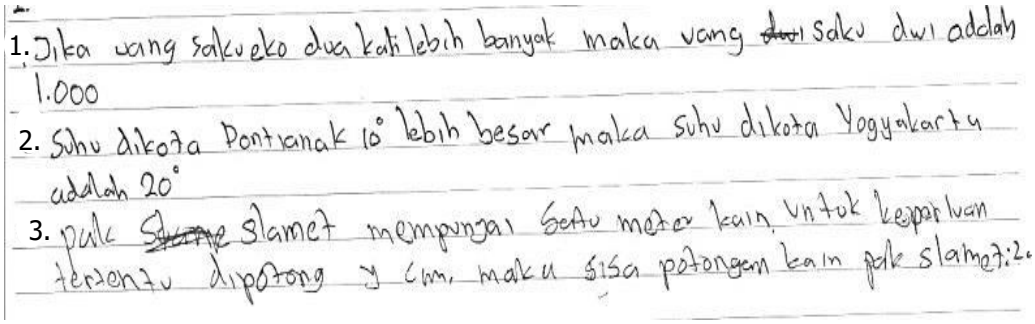
1. *Given: Eko's pocket money is twice as much as Dwi's pocket money. Eko's pocket money is m*
   Question: Dwi's pocket money.
   Answer: Dwi's pocket money is twice less than Eko
   Dwi $= 2x - m$
2. *Given: The temperature in Pontianak* $= 10° >$ *the temperature in Yogyakarta. The temperature in Pontianak is h.*
   Question: the temperature in Yogyakarta.
   Answer: Yogya $= x < 10°$
3. *Given: Pak Slamet has one meter of cloth. For certain purposes, cut y cm.*
   Questioned: the remaining cloth
   Answer: 1 m-y cm

Figure 2. FSE-US student's answer

In Table 2, there are 28% of students have wrong answers. Most FSE-NUS students cannot answer and change the symbol or variable n to a certain number. A sample answer in Figure 3 shows that students can generalize the pattern but cannot use symbols to generalize a situation and change the symbol into a certain number. The following is a transcript of the interview with the FSE-NUS student.

Researcher	:	*Do you understand the given question?*
FSE-NUS student	:	*A little, what is the meaning of* m*?*
Researcher	:	m *is Eko's money*
FSE-NUS student	:	*How many?*

Researcher	:	*m is unknown. Why is your answer like this?*
FSE-NUS student	:	*Eko has 2000, which means Dwi has 1000*
Researcher	:	*How about the other problems?*
FSE-NUS student	:	*I think the same as before, the temperature in Yogyakarta10, so the temperature in Pontianak 20, if the last question, the cloth is not cut 80 cm, meaning the rest 20.*



Translation:
1. *If Eko's pocket money is twice as much, then Dwi's pocket money is 1000*
2. *The temperature in Pontianak city is 10º higher, so the temperature in Yogyakarta is 20º*
3. *Mr. Slamet has one meter of cloth, for certain purposes, it is cut in cm, so the students of Mr. Slamet's cloth are 20*

Figure 3. FSE-NUS student's answer

Based on the interview, the FSE-NUS student does not understand what is meant by variables like m, h, and y, so they replace it with a certain number. Thus, the student can solve the problem if m, h, and y are known in value. The students do not understand the meaning of variables well (Drijvers, Goddijn, & Kindt, 2011). Many students have difficulties operating with variables in mathematical problems. Students have, not only, to identify key parts of the problems but also the underlying relationships (Dindyal, 2004). Several studies considered the students' difficulty due to the need to make sense of the problem situation and generate symbolic representations (Kieran, 2007; Koedinger & Nathan, 2004; Rakes et al., 2010). To form a statement or equation, students need to sort through the problem description to identify what quantities influence the solution and how these quantities are related (Lepak et al., 2018). However, based on the interview, students knew the relationship between quantities but did not know how to form the variable's statement. Thus, *students' difficulties in statements and equations that contain an unknown quantity are that they cannot make the correct relationship about the given situation because they do not understand the relationships of various objects/contexts and the meaning of variables.*

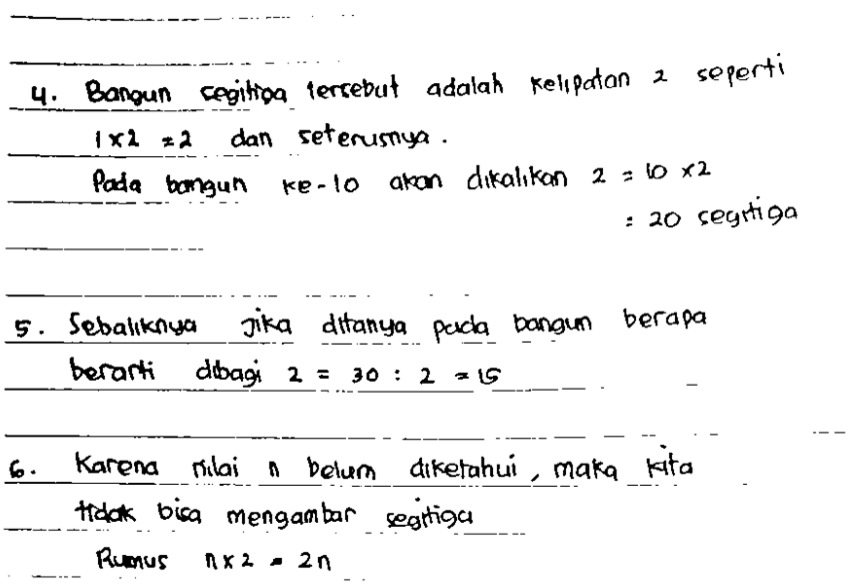
Generalizing statements from patterns

Problem 4-6 aims to generalize statements from patterns. In specific, problems 4 and 5 are to determine students' ability to find and predict patterns, while problem 6 is used to determine students' ability to generalize the pattern to the n^{th} term. The summary of students' answers to the problems is as follows (Table 3). It shows that 1 or 2 students had correct answers by using

algebra (Figure 4). It means that the students are able to determine a particular series with a pattern that has been found. It is shown in Table 3 that just 30% of the students can find the n^{th}.

Table 3. Students' answers on problems 4 to 6

Code	Problem 4 (%)	Problem 5 (%)	Problem 6 (%)	Average (%)
GP-CA	2	1	30	11
GP-CANUA	84	84	0	56
GP-WA	14	15	54	28
GP-NA	0	0	16	5



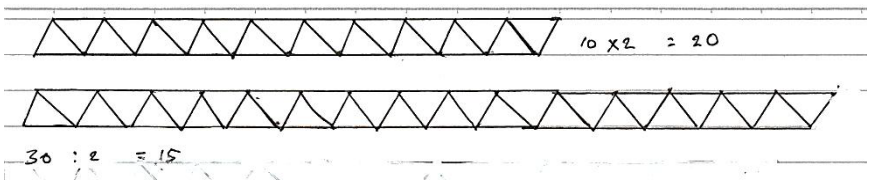
Translation:

4. *The triangular shape is multiplied by 2, such as* $1 \times 2 = 2$ *and so on. In the* 10^{th} *arrangement will be multiplied by 2* $= 10 \times 2 = 20$ *triangles*
5. *Conversely, if asked in what arrangement, it means divided by 2, then* $30: 2 = 15$.
6. *Because the value of n is not known, we also cannot draw the triangle. The formula is* $n \times 2 = 2n$

Figure 4. GP-CA student's answer

In generalizing patterns, GP-CA students' generalization types are explicit where to find the 10^{th} term. They used 2×10 without finding the next pattern. Furthermore, the GP-CA students were already in the symbolic generalization stage; students could use symbols to show the generalization of a given problem or use expressions containing variables without thinking about a particular number or a number represented by a particular letter. Symbols are usually used to represent an uncertain number in a problem (Radford, 2014). The symbol for a given number of unknowns or a quantity that varies is called a variable. Variables as a tool for generalizing expressions (van de Walle & Folk, 2008).

Table 3 also shows that more than 84% of students are able to answer numbers 4 and 5 correctly, but most of the students can answer correctly without using algebra. The students generalize the pattern using images or numbers (Radford, 2003), as shown in Figure 5.



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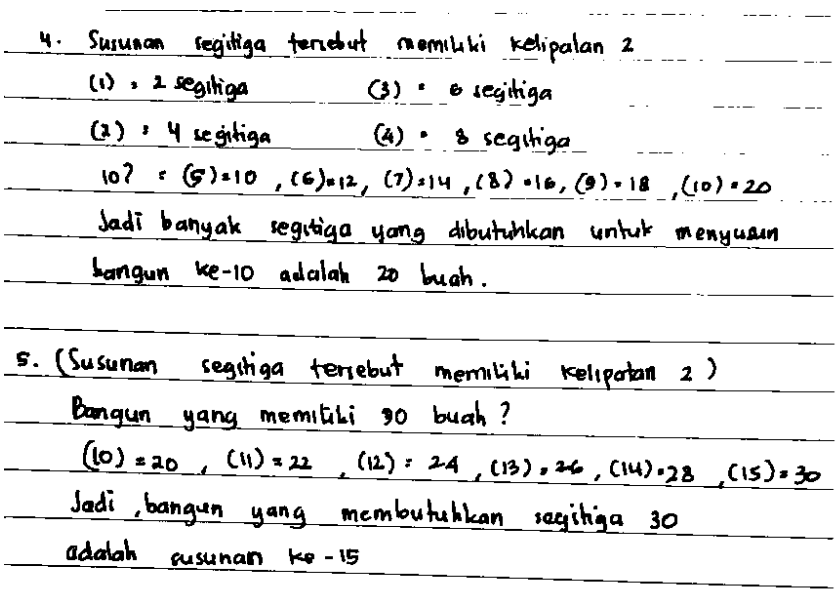


Translation:

4. *The arrangement of the triangles is multiplied by 2*
   (1) = 2 triangles (3) = 6 triangles
   (2) = 4 triangles (4) = 8 triangles
   10?= (5)=10, (6)=12, (7)=14, (8)=16, (9)=18, (10)=20
   Thus, the number of triangles needed to make the 10th arrangement is 20 triangles
5. *(The arrangement of the triangles is multiplied by 2)*
   The arrangement that has 30 triangles?
   (10)=20, (11)=22, (12)=24, (13)=26, (14)=28, (15)=30
   Thus, the shape that requires 30 triangles is the 15th arrangement

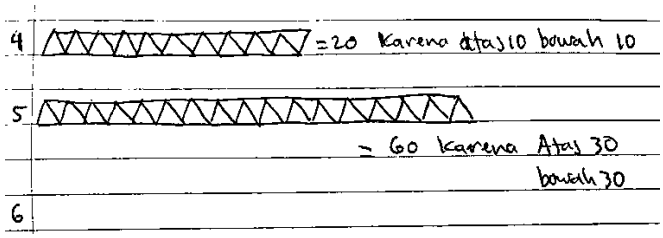
(2)

Figure 5. GP-CANUA students' answers

GP-CANUA students have been able to generalize the existing pattern by analyzing the pattern in the first arrangement until the 4th arrangement to determine the existing pattern and conclude that the pattern is a multiple of two and can predict the next pattern. In particular, El Mouhayar and Jurdak (2016) show that students regularly use the recursive strategy (pointing the common distinction among pairs of consecutive terms and repetitively adding the constant from term to term to increase the pattern) and functional strategy (relating parts of the pattern to

the figural step number) to generalize patterns. Thus, GP-CANUA students are in the contextual stage where they use images to solve problems (Radford, 2003).

About 28% of the students answered incorrectly (GP-WA), as shown in Figure 6.



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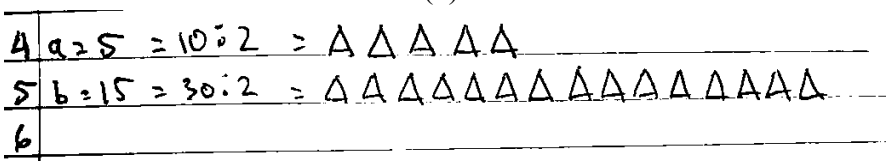

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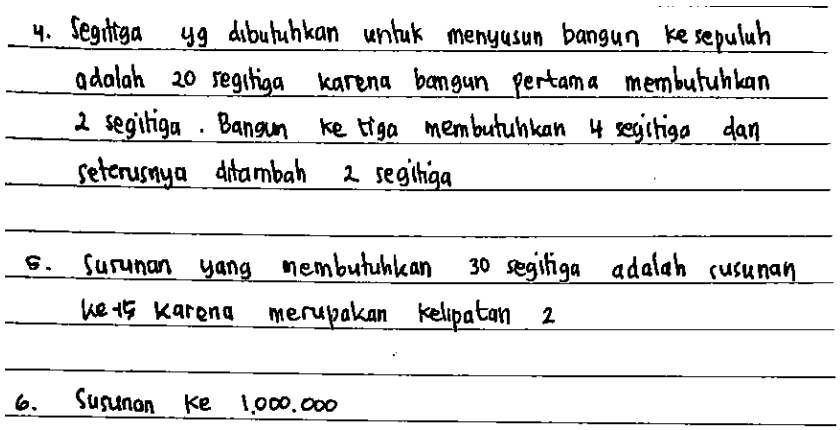

Translation:

4. *The triangle needed to construct the 10th shape is 20 triangles because the first one needs 2 triangles, the third one needs 4, and so on, plus 2 triangles.*
5. *An arrangement requiring 10 triangles is the 15th arrangement because it is a multiple of 2*
6. *1000000th order*

(3)

Figure 6. GP-WA students' answers

Students misunderstood the given patterns. The number of triangles in each shape forms a pattern multiplied by 2. However, in Figure 6 (1), students answer that the pattern in questions no. 4 and 5 are the same, namely multiplied by 2, and in Figure 6 (2), some students answer that the number of triangles forms a pattern divided by 2 form number 4 and 5. Thus, students can answer correctly for number 4 but incorrectly on number 5, or vice versa, because students do not read the questions given well. Most students think that what numbers 4 and 5 are asking is the same thing. In addition, students answer the questions given by using alphanumeric symbols

and the recursive type, which uses the previous term to find a certain term. It is shown in the following interview results.

Researcher	:	*Do you understand the questions given?*
GP-WA student	:	*Yes*
Researcher	:	*How did you find the answer to no. 4?*
GP-WA student	:	*I draw it. The first build is 2 triangles; the second build does not add two triangles. The third build does not add two more triangles; until you meet the tenth arrangement, there are 20 triangles.*
Researcher	:	*How about number 5?*
GP-WA student	:	*It is the same. I draw until the 30th arrangement. There are 60 triangles*
Researcher	:	*How about number 6?*
GP-WA student	:	*What is n?*
Researcher	:	*n is an unknown number*

On the other hand, almost 54% of students gave wrong answers to question number 6. The excerpt of the interview unravels that GP-WA student has difficulty in determining the n^{th} number pattern. The students are confused about what is meant by variable n. Drijvers et al. (2011) assert that this is one of the main difficulties in algebra. GP-WA students have predicted the next number by stating the process in words. However, when the students are asked for the n^{th} pattern, they feel confused and ask, "what is n? I do not know". *Students tend to replace n with another number and do not even answer the problem. They assume* n *is "something" which can be replaced by any number regardless of the meaning of* n. Thus, *students' difficulty generalizing statements from patterns is to generalize the pattern to the* n^{th} *term*.

The findings of this study imply that teachers must pay attention to generational activities because most of algebra's concepts are about generalization. Several studies aim to improve student's abilities in the activities using realistic mathematics education (RME) (Dani, Pujiastuti, & Sudiana, 2017; Kusumaningsih et al., 2018). The stages in RME relate to indicators of mathematical generalization ability, namely concluding (generalizing) various knowledge, facts, and experiences given to students through examples of several cases in real life to rediscover mathematical ideas and concepts. Another study (Dahiana, 2010) found that the inductive approach improved students' generalization ability. The learning process with an inductive approach begins with introducing concrete cases to abstract forms, from specific examples to general formulas (Aisyah, 2016). This process is in line with the ability to generalize. In addition, there is a learning cycle (Toheri & Winarso, 2017) that can improve overall algebraic thinking skills. In the learning cycle, there are stages of Engagement, Exploration, Explanation, Elaboration, and Evaluation that will enable students' thinking to succeed in algebra. The stages allow students to interact with peers in building knowledge dynamically and constructivism views in the acquisition of knowledge (Toheri & Winarso, 2017). Basically, according to Blanton and Kaput (2003), teachers must find ways to support algebraic thinking by creating learning where students can model, explore, debate, predict, guess, and test.

Conclusion

This study found that the 7^{th}-grade students can generalize statements from geometry patterns or sequences of numbers and numerical rules better than making statements and

equations that contain an unknown quantity representing a problem situation. The students' identified difficulties in the generational activity of algebraic thinking are understanding the problems and turning them into mathematical forms. These are due to the students' incomprehension of the relationships of some conditions and the meaning of variables in the given problems. The students also have difficulties in generalizing the patterns to the n^{th} term since they did not understand the meaning of variables. We believe that these findings provide valuable insight and entry points to design a learning activity that facilitates students' difficulties in the generational activity.

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