Beta: Jurnal Tadris Matematika, 13(2) 2020: 152-167
DOI 10.20414/betajtm.v13i2.400

Visuospatial reasoning of eighth-grade students in solving geometry problems: A gender perspective

Kartika Sulistiya Nuriswaty[1], **Sadrack Luden Pagiling**[2], **Nurhayati**[3]

Abstrak: Penalaran visuospasial sangat diperlukan dalam menyelesaikan masalah matematika terutama pada masalah geometri. Namun, banyak siswa yang mengalami kesulitan dalam melakukan penalaran visuospasial. Penelitian kualitatif ini bertujuan mendeskripsikan penalaran visuospasial siswa kelas VIII yang ditinjau dari perbedaan gender dalam menyelesaikan masalah geometri. Satu siswa laki-laki dan satu siswa perempuan dengan kemampuan matematika tinggi dipilih sebagai subjek. Tes dan wawancara digunakan untuk mengumpulkan data. Tes digunakan untuk menyelidiki penalaran visuospatial siswa dan wawancara bertujuan mengonfirmasi dan menggali lebih dalam penalaran siswa. Data hasil tes dan wawancara dianalisis dalam tiga tahap, yaitu reduksi data, penyajian data, dan penarikan dan verifikasi simpulan. Hasil penelitian menunjukkan bahwa penalaran visuospasial siswa laki-laki dan perempuan dalam menyelesaikan masalah geometri berada pada jenjang sintesis. Namun, terdapat perbedaan pada jenjang sintesis dalam mengidentifikasi keterkaitan spasial antar objek-objek. Siswa laki-laki mengutarakan informasi dari skema pandangan secara menyeluruh, sedangkan siswa perempuan mengutarakan bagian per bagian dari skema pandangan tersebut.

Kata kunci: *Penalaran visuospasial, Masalah geometri, Gender*

Abstract: Visuospatial reasoning is indispensable in solving mathematical problems, especially geometry. However, many students face difficulty in visuospatial reasoning. This qualitative study aims to describe eighth-grade students' visuospatial reasoning in solving geometry problems in terms of gender differences. One male and one female student with high mathematics ability were involved. A test and interview were utilized to collect data. The test was used to investigate students' visuospatial reasoning and the interview was administered to confirm students' reasoning. Students' test and interview results were analyzed in three stages: data condensation, data presentation, and conclusion drawing and verification. This study found that both male and female students' visuospatial reasoning in solving the problems is at the synthesis level. However, at the synthesis level, when identifying the spatial relationships between objects, the male student expresses information from the overall view scheme, while the female student expresses part by part of the view scheme.

Keywords: *Visuospatial reasoning, Geometry problems, Gender*

A. Introduction

At the secondary school level in Indonesia, the teaching and learning of geometry emphasize students' understanding, which contains mathematical objects in the form of facts, concepts, and principles found in everyday life (Sulaiman, 2019). The main objective of learning geometry is

[1,2,3] Universitas Musamus, Jln. Kamizaun Mopah Lama 99611, Merauke, Indonesia, pagiling_fkip@unmus.ac.id

to support students' understanding of spatial concepts and procedures to solve spatial problems encountered in their daily lives (Susilawati, Suryadi & Dahlan, 2017). Geometry problems are situations related to geometry presented in non-routine problems so that students cannot immediately find a solution (Afandi, 2016).

Students learn cubes and cuboids at the lower secondary school level as one of the geometry contents (Owens, 2015a). However, most eighth-grade students experienced didactical obstacles in solving cubes and cuboids problems based on their spatial perception ability (Nurjanah & Juliana, 2020). Furthermore, many students underwent difficulties in understanding geometric concepts and solving geometric problems due to their visual-spatial skills (Konyalioğlu et al., 2012). One of the reasoning forms applied to solve the geometrical problem is visuospatial reasoning (Harris, Logan, & Lowrie, 2020; Owens, 2020). This reasoning is needed, utilized, and developed by students to learn geometry. When students imagine geometric objects, they understand the similarities and differences of objects from one another or construct a geometric shape by arranging the shape's components, for example, sides, angles, planes, edges, or diagonals. Students need to develop their visuospatial reasoning since it is widely used in architecture, mechanical engineering, mathematics, and other domains (Kho, 2015; Owens, 2015b).

The notion of spatial ability refers to the ability to develop, recall, retrieve, and transform well-structured visual images (Battista, 1990, 2007). One's ability to recognize, generate, inspect, operate on, and reflect on spatial objects, images, relationships, movements, and transformations are referred to as spatial reasoning (Battista, 1990, 2007). Meanwhile, visuospatial reasoning is the mental process of forming images and concepts and mentally modifying and analyzing those visual images (Owens, 2015b). In a mathematical context, visuospatial reasoning is characterized by the incorporation of visual imagery in problem-solving. The visuospatial imagery concept includes the relationship, position, and movement of individual image elements or image sequences (Owens, 2015b). Thus, visuospatial reasoning incorporates a wide range of spatial abilities, spatial skills, visual and spatial imagery, representations and processes, and related concepts. In other words, visuospatial reasoning encompasses spatial abilities but is much broader than these skills (Owens, 2015b).

Linn and Petersen (1985) differentiate three types of visuospatial skills, namely spatial perception (identifying spatial relations concerning one's orientation), mental rotation (mentally rotating a 2D or 3D object), and spatial visualization (processing complicated multistep manipulations of spatial information). The term visuospatial reasoning emphasized the reasoning associated with and dependent on visual imagery and expressed and argued with spatial references (Owens, 2015b). Visuospatial reasoning is a particular skill that uses visual images and visual relationships in solving problems (Kho & Tyas, 2018; Owens, 2020), and in understanding mathematics, someone needs to have visuospatial reasoning (Hunt, 1995). Visuospatial reasoning is a thought process related to making conclusions about visual information related to the spatial relationships between objects (Kho, 2015). In this study, it comprises three aspects, namely analysis, synthesis, and conclusion drawing adapted from Kho (2015) and Owens (2015b).

Researchers have documented that visuospatial reasoning has a positive relationship with mathematics performance (Cheng & Mix, 2014; Lowrie et al., 2017; Owens, 2015a). Cheng and Mix (2014), who investigated children aged 6-8 years, proved that spatial training could improve mathematics achievement. Lowrie and colleague's study scrutinized that class-based spatial reasoning interventions can improve elementary school students' mathematics performance (Lowrie et al., 2017). Therefore, students who have sound visuospatial reasoning are believed to be proficient in solving mathematics problems. Improving spatial skills could be a neglected strategy to improve student performance on large-scale tests and ultimately better prepare children for successful adulthood (Sorby & Panther, 2020). Several scholars also argue that improving visuospatial thinking could help provide the skills essential to succeed in mathematics (Miller & Halpern, 2013; Sorby & Veurink, 2019; Uttal et al., 2013). Miller and Halpern (2013), who investigated the benefit of spatial training among 28 female and 49 male undergraduate students, emphasized that the spatial training group improved over time in the skills needed to rotate and visualize cross-sections of 3-D objects mentally. Sorby and Veurink (2019) found that grade 7 students who participated in their spatial skills training program performed better on both state-wide mathematics assessment and local placement tests for 9th-grade. From the previous studies, it can be concluded that visuospatial reasoning plays a paramount role in teaching and learning mathematics in general and geometry in specific.

Other factors, such as gender, can also influence reasoning ability (Battista, 1990; Habacha et al., 2014; Harris et al., 2020; Linn & Petersen, 1985; Reinhold et al., 2020). The differences in the thinking processes of men and women occur after going through various experiences. In solving a similar problem given to several individuals, each individual will undoubtedly respond differently. There are differences in thinking patterns between men and women in doing something related to these differences in structure, such as emotions, behavior, language methods, spatial abilities (spatial), and mathematic problems (Susilowati, 2016). The spatial ability of men in solving the problem of flat-sided shapes has shown an ability that meets the aspects of spatial ability. Female students' spatial abilities only fulfill several aspects of spatial abilities (Purborini & Hastari, 2018). In mathematical reasoning, female students are better than male students in solving geometric reasoning problems (Salmina & Nisa, 2018).

Battista (1990) showed that gender differences are near related to geometric thinking and spatial ability, affecting student achievement of geometry. Moreover, Battista suggested that teaching and learning geometry should pay attention to gender to understand how students learn geometry better. Zhu (2007), who examined gender differences in solving mathematics problems, indicates differences in mathematical problem solving, which are influenced by gender differences associated with cognitive abilities influenced by psychological traits, experience, and education. Male students are generally superior in spatial and arithmetic abilities, while female students are generally superior in verbal ability. Furthermore, another study pointed out that adult males show much higher spatial abilities than adult females (Reinhold et al., 2020).

The relationship between visuospatial reasoning and gender difference has been well documented (Casey, 2015; Hegarty, 2017; Reinhold et al., 2020). However, there is less research on how students cope with geometric problems using mental activity to conclude new objects from three perspectives; recognizing shapes from above, front, sides (Febriana, 2015; Kho, 2015; Kho & Tyas, 2018). In the current study, the mental activity in visuospatial reasoning includes analyzing visual information, synthesizing visual information, and drawing conclusions about the final shape of the formed objects. Analyzing an object's visual information refers to making observations of the object's parts, determining how one part relates spatially to another or the entire structure. Synthesizing visual information from an object means combining elements and spatial relationships between elements to form an object. Drawing conclusions about the final shapes refer to mental activity to determine the internal representation of the object formed. There are plenty of ways that can be done to build shapes in three dimensions (Kho, 2015). One way is to use three-view scheme from the front, the right side, and above. The view can be traced as a schematic, and the schematic is called a schematic view. A schematic view of an object is an image that the object resembles from the front, right side, and top. A description of students' visuospatial reasoning in constructing the geometry shapes based on the schematic view is the focus of this study.

It is essential to explore the profiles of students' visuospatial reasoning in terms of gender. Acknowledging that spatial skill is not immutable, learning and practicing can enhance an essential first step for any targeted ability intervention to remove the gender gap and ensure gender equity. Moreover, by profiling the visuospatial reasoning, the teachers will recognize didactical situations and students' obstacles regarding individual differences, especially gender, in geometry teaching and learning. Therefore, the current study provides a lens on male and female students' visuospatial reasoning in solving geometric problems.

B. Methods

Since we explored students' visuospatial reasoning on a small-scale, we consider a qualitative study as a representative approach. This study was conducted at one of the public lower secondary schools in Merauke, Indonesia, during this pandemic time. The participants were two students purposively selected from twenty-eight students based on students' high mathematics ability using their current achievement report and the mathematics teacher's suggestions. We applied homogenous sampling (Miles, Huberman, & Saldana, 2014) since the participants have similar demographics and characteristics. The selection of participants was planned to use a test, but it switched to the student report due to the Covid-19 situation.

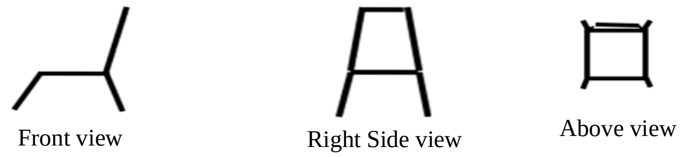
The instruments used in this study were the visuospatial reasoning test on geometric problems and interview guidelines. The test was adapted from Owens (2015b), which demands students to recognize shapes from different perspectives. We employed this test because it includes objects such as chairs, cameras, and swings that are familiar to students. These instruments have gone through construct validation by two mathematics education experts.

The participants worked on the given problems, and then a semi-structured interview was conducted. We administered a semi-structured interview because it provided flexibility to pose questions that are not planned if clarification is needed (Zazkis & Hazzan, 1998). The questions refer to the level of visuospatial reasoning made up of analysis, synthesis, and conclusions. The participants were asked to talk in detail about their works in solving the problems. Data triangulation was assigned to obtain data credibility. Visuospatial reasoning tests and interviews were carried out twice to obtain credible data. After twelve days, the second test and interviews were administered to the participants. The first test and interview were compared with the second part to meet the indicators in Table 1.

There were four geometry problems given to the participants that would evoke discussion on how they reason visually and spatially.

VsRT 1, Number 1.

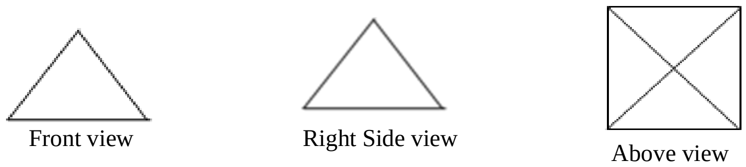
There are three schematic views of a figure model, as shown below.



By paying attention to the three-view schema above, could you try to describe the shape?

VsRT 1, Number 2

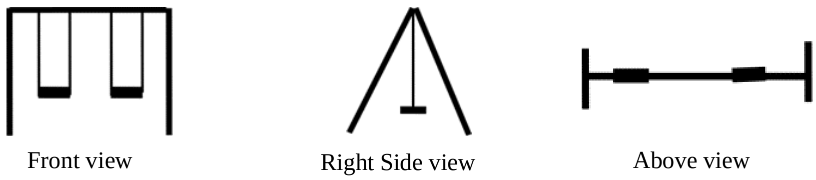
There are three schematic views of a figure model, as shown below.



By paying attention to the three-view schema above, could you try to describe the shape?

VsRT 2, Number 1

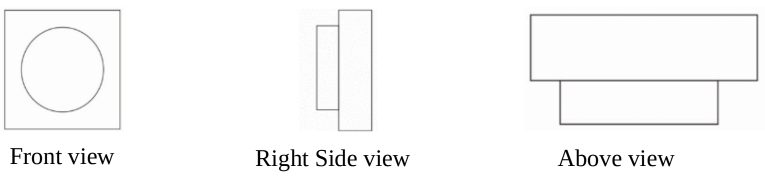
There are three schematic views of a figure model, as shown below.



By paying attention to the three-view schema above, could you try to describe the shape?

VsRT 2, Number 2

There are three schematic views of a figure model, as shown below.



By paying attention to the three-view schema above, could you try to describe the shape?

Data analysis followed three stages; condensating data, displaying data, and drawing and verifying conclusions (Miles et al., 2014). We firstly compared students' answers on the test to examine the consistency. Then transcripts of the interviews were made and compared as well. Once the consistency of the data was achieved, we coded students' answers and transcripts referring to the indicators of visuospatial reasoning (Table 1). Coding the data is an essential part of the data condensation. In displaying the data, the method of describing variability with a conceptually clustered matrix (Miles et al., 2014) was employed. In this case, each participant's condensed answers and interview were placed in the matrix so that the fulfillment of the indicators of visuospatial reasoning can be easily identified. For this manuscript, we only displayed students' answers in figures and excerpts of interviews. Based on the matrix, conclusions regarding each student's profile of visuospatial reasoning were drawn and verified. In verifying the conclusions, investigator triangulation was carried out (Rothbauer, 2008). All authors were actively involved in data collection and verification of the findings to reach a consensus conclusion.

The indicators of students' visuospatial reasoning in solving the problems presented in Table 1 are drawn from Kho (2015) and Owens (2015b).

Table 1. Indicators of visuospatial reasoning

Level	Indicators
Analysis	(1) Students can observe and identify an object based on the information provided, (2) identify the spatial relationship between objects, and (3) associate and transform between objects from two to three dimensions.
Synthesis	(1) Students can observe and identify an object based on the information provided, (2) identify spatial relationships between objects, (3) associate and transform objects from two to three dimensions, and (4) combine elements and spatial relationships between objects to form an object new.
Conclusion drawing	(1) Students can observe and identify an object based on the information provided, (2) identify spatial relationships between objects, (3) associate and transform objects from two to three dimensions, (4) combine elements and spatial relationships between objects to form an object new, and (5) conclude a new object accurately and correctly obtained from the fusing process.

C. Findings and Discussion

This section presents each participants' works on the visuospatial reasoning test and the excerpt of interview transcripts which indicate their analysis, synthesis, and draw conclusions. We then interpret the data to formulate the present study's findings, as summarized in Table 2.

Male student's visuospatial reasoning

The male student's (MS) solution and interview excerpts in solving VsRT 1 and VsRT 2 are as follows.

Transcript 1 (VsRT 1 number 1)

R : *What actions are you taking to address this issue and compose the required shapes?*
MS : *(Thinking for a minute..) First of all, do this first (pointing to a front image). What is it called? First, look forward, then look sideways, then adjust to the top (pointing to the image in the graphic view).*
R : *What picture do you see?*
MS : *Like a chair*
R : *What is the shape of the chair? Maybe you can be more specific in shape*

MS : Hmmmm.... (thinking for a minute..). Like this (pointing to the image he constructed)
R : Oh, so what you imagined is similar to what you have drawn?
MS : Yes, they are the same

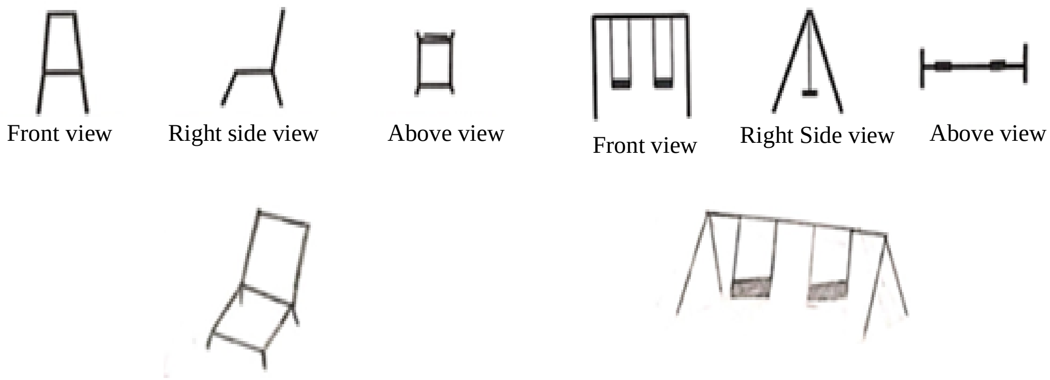


Figure 1. MS' answer on number 1 of the VsRT 1 and VsRT 2

Figure 1 and transcript 1 show that MS could observe and identify an object based on the information provided. MS explained what is known on the problem; the shapes are seen from the front, right side, top view, and associated the known image with the unknown on the problem, which is to describe a chair image. However, MS was not able to communicate in detail about what he has imagined. He did not know the chair's representation exactly whether it is the same as the right image or not. Consequently, MS was unable to conclude a new object accurately obtained from the synthesis of the three-view schema.

Transcript 2 (VsRT 2 number 1)

R : Do you know what is being asked in this problem?
MS : Yes
R : What information do you get from this three-view schema?
MS : It looks like a swing
R : So, from the three schemes of this view, in your mind, is it like a picture?
MS : Swing
R : Whether the description in the schematic drawing of the view given is sufficient to solve this problem?
MS : Enough (answering convincingly)
R : What dimensions are the final results for the shapes that will be formed?
MS : Three dimensions
R : Could you explain the steps in solving this problem to compose the required shapes?
MS : Firstly, draw the top (pointing to the image he made) is the same swing, then the right and left are adjusted.
R : What kind of image do you think?
MS : Like a swing

Figure 1 and transcript 2 prove that MS was able to pinpoint and retrieve an object based on the details given. He restated what is known in the problem, that is, the shape looked from the front, right side, top, and correlated the known image with the unknown on the problem, which is to describe the swing image. MS described that at first, he looked at the front view schema, then adjusted it to the right and top to create a new shape. Subsequently, he imagined the shape that will be formed is the swing. However, the representation built by MS was not compatible with the exact image of the final shape. The incompatibility of the MS' image with the right image lies in the pole and the swing seat.

The data (Figure 1, transcripts 1 and 2) uncover that the male student's visuospatial reasoning is consistent. He could observe and recognize an object based on the given information. In solving VsRT number 1, MS explained the problem: the front view, right side, and the same top and associates the known image with the unknown from the problem, which describes a shape. He could identify spatial relationships between objects. Indeed, MS explained that the three schematic views resemble shapes, and the description of the schematic views of the image is sufficient to solve the problem. Furthermore, MS linked and transformed objects from two dimensions to three dimensions. For number 1, MS explicated that the shape will be 3D shapes and the object's name from the image. Subsequently, MS managed to combine elements along with the spatial relationships of objects to form a new object. He firstly looked at the front, right, and top view of the picture to create a new shape. However, MS was unable to conclude a new object accurately obtained from synthesizing the three-view schema. The representation developed by MS was in line with the exact image of the final shape. Thus, MS's visuospatial reasoning was at the synthesis level.

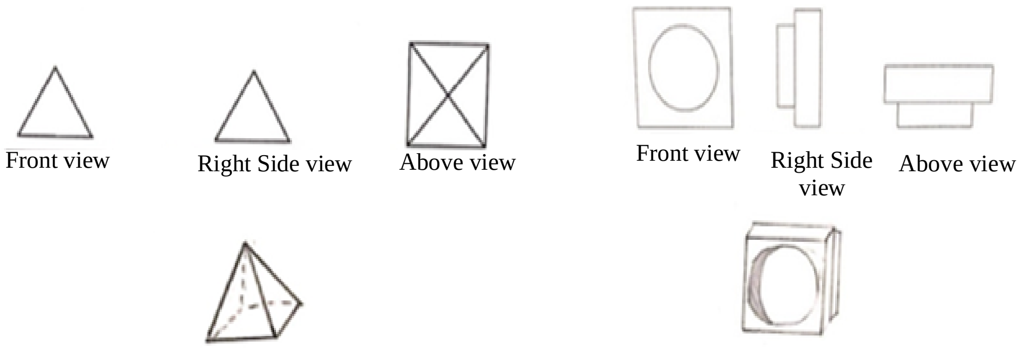


Figure 2. MS' answer on number 2 of the VsRT 1 and VsRT 2.

Transcript 3 (VsRT 1 number 2)

R : What do you know from the questions?
MS : Front view, right side, and top view of the picture
R : Then, do you know what was asked by the problem?
MS : Yes, I know (answering convincingly)
R : Then, what information is obtained from the three-view schema?
MS : The shape of the model is the same as a prism
R : Sure? Come on, try to see it again. Is this a prism or not? (think and pay attention to the picture he made).
MS : Uhmm, it is wrong. It is not a prism but a pyramid
R : So, the name of the shape was incorrect?
MS : Yes, it should be a rectangular pyramid
R : Then, whether the description in the schematic drawing of the view given is sufficient to solve this problem?
MS : It is enough

R	:	*What dimensions are the final results for the shapes that will be formed?*
MS	:	*Three dimensions*
R	:	*How did you solve this problem to construct the required shapes?*
MS	:	*Firstly, I made a three-sided triangle, then drew the base, then constructed the lines (pointing to his constructed picture) to make the dimensions*
R	:	*What image do you see?*
MS	:	*Like a pyramid*
R	:	*What is the shape of the pyramid? Maybe, can you explain in more details?*
MS	:	*Em (thinking for a moment..) as I describe it (pointing to the picture he made)*
R	:	*So, your representation and the results of your picture are correct?*
MS	:	*Yes, something like that*

Figure 2 and transcript 3 show that MS could collect information and identify an object based on what is provided. In addition to explaining information on the problem, MS detailed what is known from the problem; schematic view from the front, the right side, the top view, and associated the known image with the unknown on the problem. Initially, he mentioned the shape is a prism. When looking at the three-view schema carefully, he concluded that the shape is a pyramid. MS described that he constructed three sides of the triangle, drew the base, and constructed the lines. Afterward, he imagined the shape that will be formed as the pyramid. However, MS's representation and the solution did not suit the exact image of the final shape. Based on the MS' answer in Figure 2, there is a discrepancy between the subject's image and the right side view scheme.

Transcript 4 (VsRT 2 number 2)

R	:	*What information do you get from the three-view schema?*
MS	:	*Front view, appears right side, and top view*
R	:	*The schematic view given is sufficient to solve this problem?*
MS	:	*Enough (answered convincingly…).*
R	:	*What dimensions of the final shapes will be formed?*
MS	:	*3D shapes*
R	:	*What does the image look like in daily life?*
MS	:	*I think if I am not mistaken, like that camera*
R	:	*How do you compose the required shape?*
MS	:	*The first image looks at the front, and new adjust to the sides*
R	:	*What do you imagine?*
MS	:	*Like this, adjusting it under the camera is difficult (pointing to his picture), so it is a bit confusing*

Figure 2 and transcript 4 show that MS observed and identified an object based on the given information. MS explained what is known from the problem; the view from the front, right side, top view, and linked the known image with the unknown on the problem. MS described that firstly, he looked carefully at the front view schema and adjusted to the sides. Then he imagined the shape that will be formed as a camera. However, MS's representation and solution were not compatible with the exact image of the final shape. Based on the MS' answer on number 2 of the VsRT 2 (Figure 2), there is also a discrepancy between the subject's image and the right-side view scheme. He got confusing in drawing the camera accurately.

Figure 2, transcript 3, and transcript 4 show consistency in male student's visuospatial reasoning. MS was able to examine an object and identify it based on the provided details. As in the case of VsRT number 2, he explained what is known about the problem: The front view, right side view, and the top view and associated the known image with the unknown shape existed the problem. He was capable of determining spatial relationships between objects. He explained that the three schematic views were comparable to shapes and further explained that it was sufficient to describe the view's schematic image to solve the problem. MS associated and transformed 2D objects into 3D objects. On number 2, MS explained that the shape would be 3D shapes and the object's name from the image. Subsequently, MS combined elements along with the spatial relationships of objects to form a new object. He firstly looked at the front, right, and top view of the picture to create a new shape. However, MS is unable to conclude a new object accurately obtained from synthesizing three-view schema. The representation made by MS is not compatible with the exact image of the final shape. Thus, MS's visuospatial reasoning is still at the synthesis level.

Female student's visuospatial reasoning

The following are the female student’s (FS) answers and excerpts of interviews in solving number 1 of VsRT 1 and VsRT 2.

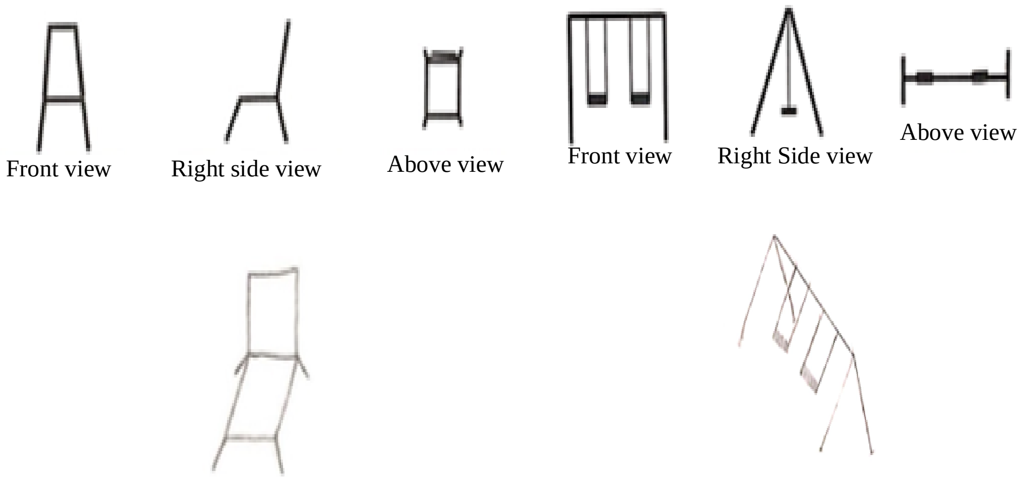


Figure 3. FS' answer on number 1 of the VsRT 1 and VsRT 2

Transcripts 5 (VsRT 1 number 1)

R : How did you compose the required shapes?
FS : Hmmm. First, look at the image, scrutinize it, the front view, the side view, and how the top is. Continue to imagine the shape, then draw it.
R : What picture do you think?
FS : Em (think for a moment…) like a chair
R : How about that? Could you explain the shape more specifically?
FS : Yes, like an ordinary chair
R : Yes, usually like what?
FS : Em, like this (pointing to her constructed picture)
R : Oh, so the picture you thought is the same one you represent?
FS : Yes, something like that

Figure 3 and transcript 5 show FS made observations and identification on an object based on the information. She looked at the schematic view from the front, right side, and top view and then related the known image with the unknown on the problem to imagine an ordinary chair. Furthermore, FS could combine elements along with spatial relationships between objects to form a chair. However, she did not know the chair's representation exactly whether the same as the right image. Figure 3 reveals a discrepancy between the FS's image and the scheme's front view and top view. Consequently, she could not conclude a new object accurately obtained from synthesizing the three-view schema.

Transcript 6 (VsRT 2 number 1)

R : What information do you get from the three-view schema?
FS : Em. The front view of the shape, the side view, as well as the top view
R : Is the description in the schematic view given sufficient to solve this problem?
FS : Enough (answered convincingly…)
R : What dimensions of the final shapes will be formed?
FS : Three dimensions
R : What does the picture look like?
FS : Swing
R : How do you compose the required shapes?
FS : Em. Firstly, look at the instructions of the problem carefully, pay attention to what the front looks like, what the side view is, and how the top is. Continue to imagine and draw

Figure 3 and transcript 6 uncover that FS noticed and recognized an object based on the provided information. In this case, she restated what is known from the problem; the front view of the shape model, right side view, also top view. She emphasized that the image that will be formed is a swing. In addition, she could combine elements along with spatial relationships between objects to form a new object. FS explained the instructions on the problem carefully, paid attention to the front view, side view, and top view, then imagined and described the shapes. However, she was incapable of concluding a swing accurately and correctly obtained from the synthesizing process. Figure 3 shows a discrepancy between the results of the subject's description and the schematic of the side view and top view.

It is clear from Figure 3, transcript 5, and transcript 6 that FS could make observations and identify an object based on the given information. Moreover, FS could draw the object from three distinct perspectives, albeit being not accurate. In solving number 1 of the VsRT 1 and 2, FS explained what is known from the problem: the front view of the shape, its right side view, and its top and linked the known image with the one asked in the problem. FS was able to identify the spatial relationships between objects by explaining three different view schemes, combining elements and spatial relationships between objects to form a new object. However, she could not conclude a new object accurately and correctly obtained from the synthesizing information.

Transcript 7 (VsRT 1 number 2)

R : What do you know about the problems?
FS : The front view of the shape, its right side view, also from above
R : Do you know what the problems ask you to do?
FS : Describe the shape according to what is known

R : *What dimensions of the final shapes will be formed?*
FS : *Three dimensions*
R : *Then what kind of image in your mind?*
FS : *A pyramid*
R : *How do you construct the required shapes?*
FS : *Pay attention to the problem first, keep looking for the information needed, imagine it and, keep drawing it.*

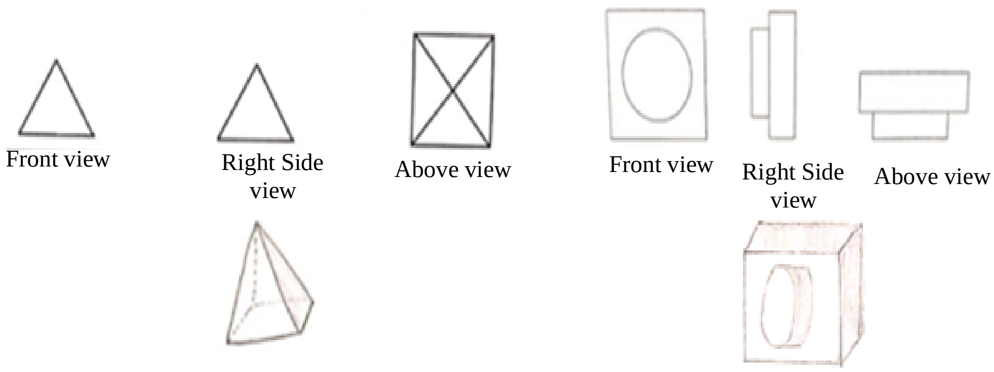


Figure 4. FS' answer on number 2 of the VsRT 1 and VsRT 2

Figure 4 and transcript 7 show that FS identified an object based on the given information by explaining what is known from the problem; front side, right side, top view, and linking the known image with the unknown on the problem. Furthermore, FS described that she looked at the problem first, kept looking for the information needed, imagining it, and then drawing it. She imagined the shape that will be formed like a pyramid. However, the representation made by FS did suit the exact image of the final shape. Figure 2 shows there is a gap between the subject's image and the right side view scheme.

Transcript 8 (VsRT 2 number 2)

R : *What do you know about the problems?*
FS : *Front view of a shape model, right side view, also above it*
R : *Do you know what is asked on the problem?*
FS : *Draw the model of the shape*
R : *What dimensions will the final shape be formed?*
FS : *Three dimensions*
R : *What does the image look like?*
FS : *Em (thinking for a moment…) I do not know what is wrong. Nevertheless, it looks like a camera*
R : *How did you arrange the shape?*
FS : *Firstly, pay attention to the information in the problem. Secondly, connect it with everyday objects, and lastly, continue drawing*

Figure 4 and transcript 8 prove that FS was able to discover and identify an object based on the available information. She presented detailed information from the problem: front side, right side, top view, and associated the known image with the unknown on the problem. Moreover, she firstly looked at the problem the connecting it with everyday objects, and continued drawing.

She visualized the shape that will be formed as a camera. However, the representation by FS is not appropriate with the exact image of the final shape. Based on the FS's solution in Figure 4, there is a gap between her constructed image and the right-side view scheme. She could not portray the camera accurately.

Figure 4, transcript 7, and transcript 8 reveal consistency in female students' visuospatial reasoning. FS observed and identified an object based on the provided information. In solving VsRT number 2, FS explained information on the problem: the front view, right side view, and the top view and made a link to the known image with the unknown from the problem. She recognized spatial relationships between objects and explicated the three perspectives resemble shapes. She also argued that the schematic view of the image is sufficient to solve it. Furthermore, she could perceive the objects from 2D to 3D shapes. In number 2, FS described the shape as 3D shapes and the object's name from the image. She synthesized the elements along with the spatial relationships of objects to construct a new object. She firstly looked at the front, right, and top view of the image to create a new shape. However, FS was unable to conclude a new object accurately obtained from synthesizing three perspectives. The representation made by FS does not quite depict the exact image of the final shape. Thus, FS's visuospatial reasoning was still at the synthesis level.

We have displayed students' solutions, interviews, and interpretations of students' visuospatial reasoning in solving geometric problems. Table 2 summarizes the findings of the present study.

Table 2. Summarized findings of students' visuospatial reasoning in solving geometry problems

Level	Indicators	Students' visuospatial reasoning	
		Male	**Female**
Analysis	*The students are able to:* Observe and identify an object based on the information provided, Identify the spatial relationship between objects, and Link and transform between objects from two dimensions to three dimensions.	√	√
Synthesis	*The students are able to:* Make observations and identify an object based on the information provided, Identify spatial relationships between objects, Link and transform objects from two dimensions to three dimensions, and Combine elements and spatial relationships between objects to form an object new.	√	√

Table 2 reveals that the male and female students have different visuospatial reasoning. The difference is not from the reasoning level but in their reasoning when looking at the view scheme. Prior findings also confirm that students’ visuospatial reasoning in solving geometric problems is different from one subject to another (Battista, 1990; Kho & Tyas, 2018). Only one direction is seen by the male student, while the female student divides her perspective to help look it all over. Thus, the female student rotates 2D objects (using her visuospatial reasoning) to bring up

3D objects in her mind. The male and female students did not use the same viewpoint when solving problems requiring 3D image rotation. Schemes of the objects play an essential role in the learning process, enabling students to identify objects and concepts without reading.

In terms of visuospatial reasoning, female students are better than male students. It is evident in the interview process where male students state the whole in looking at the three-view schema and immediately mention the object's name to be formed. The female student could know the new shape formed if she carefully looked at the three-view schema. The male student sees the three schematic views as a whole. He immediately recognizes the shape that will be formed since he correlates it with objects in daily life. It indicates that the male student depends on the physical object. On another side, the female student identifies the new object since she re-seeing the shapes carefully in different orientations and imagining parts by parts of the shapes. The female student is mentally manipulating the object and building representations in her mind. Some prior studies found that male students outperform female students in spatial abilities (e.g., Reinhold et al., 2020). On the other hand, Logan and Lowrie (2017) revealed no significant gender differences in spatial visualization abilities.

Both male and female students are less able to conclude a new object accurately. They have doubts about constructing a new shape, and the final shapes are incompatible with the three schematic views since their representations are inaccurate (Boonen et al., 2014). Thus, it can be seen that the students faced difficulties constructing 3D shapes from three distinct perspectives of the 2D shapes (Nurjanah & Juliana, 2020; Owens, 2015b). These findings imply that the mathematics teachers need to design rich geometric instructions, which support students in developing representations. This can be done, one of them, by providing relevant media such as dynamic geometry software to visualize the 2D and 3D shapes.

This study focused only on two students with high mathematics ability. Different results might be achieved if a similar study involves students with different mathematical abilities. However, we argue that the findings of this study provide an insightful lens to understand female and male students' spatial reasoning in solving the geometric problems. In the context of the mathematics classroom, which comprises students with diverse abilities in mathematics, we might find some students could not reach the synthesis level or even struggle with the first level. In this case, further research is necessary to fully understand students' profile of visuospatial reasoning with various mathematical abilities. The profiles will be a starting point for mathematics educators to design instructions, which help students develop visuospatial reasoning and solve geometric problems as well.

D. Conclusion

Visuospatial reasoning is a thought process that involves the sense of sight by manipulating and rotating an object so that it can describe the object correctly. In this study, both male and female students reach the synthesis level in visuospatial reasoning when solving geometry problems. At this level, students have been able to associate the images they see with objects in everyday life and describe these objects appropriately; this is a supporting ability in supporting students' mathematical achievement in schools. However, the process of identifying spatial relationships between objects is distinct. The male student expresses information from the overall schema of view. In comparison, the female student expressed part by part of the schematic view. The students' visuospatial reasoning explains that each individual's thinking is different from one another. The mathematics teachers should consider this in order to create rich geometric activities that foster students' visuospatial reasoning involving mental activities in making analysis, synthesis, and conclusion at the secondary schools.

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