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Defragmenting structures of students' translational thinking in solving mathematical modeling problems based on CRA framework

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Abstract: Fragmentasi struktur berpikir merupakan kegagalan konstruksi yang terjadi di dalam memori akibat dari konsep-konsep yang dipelajari tidak terkoneksi dengan baik. Hal ini membuat mahasiswa sering mengalami kesulitan dan kesalahan dalam memecahkan masalah pemodelan matematika. Untuk itu, perlu dilakukan upaya agar tidak terjadi fragmentasi struktur berpikir yang permanen. Dalam memecahkan masalah pemodelan matematika, mahasiswa perlu melakukan berpikir translasi, yaitu mengubah representasi sumber menjadi representasi yang ditargetkan. Penelitian ini bertujuan untuk mendeskripsikan upaya mahasiswa dalam melakukan penataan fragmentasi struktur berpikir translasi yang terjadi (defragmentasi struktur berpikir translasi) dalam memecahkan masalah pemodelan matematika. Defragmentasi yang dilakukan mahasiswa dipetakan melalui kerangka CRA (*checking, repairing, dan ascertaining*). Subjek penelitian adalah mahasiswa semester 4 dan 6 yang terdiri dari 3 orang dipilih dari 85 mahasiswa. Analisis data dilakukan melalui tiga tahap, yaitu pengategorian data, reduksi data, dan penarikan kesimpulan. Penelitian ini menemukan tiga jenis defragmentasi struktur berpikir translasi: defragmentasi dari representasi verbal ke grafik, dari representasi grafik ke simbol (bentuk aljabar), dan representasi grafik dan simbol (bentuk aljabar) ke model matematika. Penelitian ini menunjukkan pentingnya pengajar matematika memberikan kesempatan kepada mahasiswa dalam menata struktur berpikirnya ketika mengalami kesulitan dan kesalahan dalam memecahkan masalah matematika.

Kata kunci: *Struktur berpikir, Fragmentasi, Defragmentasi, Berpikir translasi, Kerangka CRA*

Abstract: The fragmentation of thinking structure is a failed construction existing in students' memory due to disconnections on what they have learned. It makes students undergo difficulties and errors in solving mathematical modeling problems. There is a need to prevent permanent fragmentations. The problem-solving involving modeling problems requires translational thinking, changing from source representations to targeted representations. This research aimed to formulate undergraduate students' effort in restructuring their fragmented translational thinking (defragmentation of translational thinking structure). The defragmentation was mapped through the CRA framework (checking, repairing, ascertaining). The subjects were three of eighty-five 4th and 6th-semester students. Data were analyzed through three stages; categorization, reduction, and conclusion. The analysis resulted in three types of defragmentation of translational thinking structure: from verbal representations to graph representations, from graph representations to symbolic representations (algebraic forms), and from the graph and symbolic representations to mathematical models. The finding shows that it is essential for mathematics educators to allow students to manage their thinking structures while experiencing difficulties and errors in mathematical problem-solving.

Keywords: *Thinking structure, Fragmentation, Defragmentation, Translational thinking, CRA framework*

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A. Introduction

In solving mathematical modeling problems, undergraduate students often experience some difficulties and have error answers (Seah, 2005; Yost, 2009; Dorko, 2011; Serhan, 2015). To solve the problems correctly, the students require rich ideas, strategies, and mathematics formulation. The difficulties emerge when they cannot solve the problems with routine procedures. Several studies (e.g., Tall & Razali, 1993; Zakaria, Ibrahim, & Maat, 2010; Wibawa, Subanji, & Chandra, 2013; Booth et al., 2014; Subanji, 2015; Veloo, Krishnasamy, & Wan-Abdullah, 2015) have addressed about the students' errors. However, they have not investigated the roots of the students' errors through students' thinking in solving the problems. Seah (2005) found three errors made by students in solving mathematics problems; conceptual errors, procedural errors, and technical errors. Nevertheless, Seah (2005) had not explored students' thinking in making the errors. Therefore, further research is necessary to analyze the students' thinking in problem-solving in general (Serhan, 2015) and how they produce errors in specific.

Subanji (2015) explains that when the students receive information such as concepts, procedures, or others in learning mathematics, they have successfully constructed what they have been taught. However, some are correctly constructed (concepts that are fully understood), while others are not. The latter resulted in disconnected or unmanaged information in the scheme (Subanji, 2015). It is what Wibawa et al. (2017) called fragmentation. Subanji (2016) accounts that the fragmentation of the students' thinking structure is a phenomenon of inefficiency in storing information, which obstructs the construction of concepts and problem-solving in mathematics. The term 'fragmentation' is also well known in computer science where the storage is not effectively used. This reduces the storage capacity. We argue that fragmentation of the students' thinking structure is the source of their difficulties and errors in solving mathematics problems.

The students often have a fragmented thinking structure when solving mathematical problems involving modeling activities (Subanji, 2016; Wibawa et al., 2017; Wibawa, 2019). The students need to have translational thinking that is the ability to change the source representation to the targeted representation to solve modeling mathematics problems. In this transformation of the representations, the fragmentation of thinking structure can be identified when they have errors in changing the old representations (source representations) to the new representations (targeted representations) in the form of a mathematical model (Wibawa, 2019). The construction errors were happening in making the representations and caused by confounding schemes (Bossé, Adu-Gyamfi, & Cheetham, 2011), or the schemes are ambiguously constructed because of lack of awareness in calling the existing schemes. If this is ignored, then a permanent fragmentation of the thinking structure will happen. In this case, the students will have a continuous failure in constructing mathematics concepts.

The current research focused on restructuring the students' fragmentation called defragmentation of students' translational thinking structure. Subanji (2016) explains that defragmentation refers to the changes in thinking structure caused by some interventions. The intervention in this research is limited; that is our attempt to provide a condition where the students can manage their thinking structure. In this circumstance, it is a planned or artificial defragmentation. Subanji (2016) also asserts that the planned defragmentation can be carried out in several activities; (1) providing a cognitive conflict by comparing prior knowledge with the new problems, or (2) providing scaffolding in problem-solving. Both the cognitive conflict and scaffolding are based on the students' errors. The cognitive conflict is being aware of the imbalance in the students' understanding of mathematics problems (Lee & Kwon, 2001; Maharani & Subanji, 2018; Pratiwi et al., 2019). The scaffolding can help the students face difficulties in solving problems by giving some hints or step-by-step guidance to deal with the problems independently (Anghileri, 2006; Bikmaz et al., 2010; Bakker, Smit, & Wegerif, 2015; Ormond, 2016).

The restructure of thinking is another term used to denote the defragmentation of thinking structure. Maag (2004) explains that it is a technique frequently employed to change people's less adaptive thinking. In addition, Indraswari (2012) encourages individuals to seek alternative ways of thinking when the existing one does not work. That is to say; we need some efforts to improve the errors by restructuring the thinking. Data restructure in the computer can be related to the restructure of thinking in the human brain. The process is invisible, but the outputs are observable; it can step in problem-solving (Subanji, 2011). McKay and Fanning (2000) also argue that cognitive restructuring can be done by identifying thinking errors such as self-critiques and then re-managing the thinking by refusing the critics.

The defragmentation of students' thinking structure in mathematics problem-solving is under-studied (Subanji, 2016; Herna, 2016). Subanji (2016) elucidates that the existing fragmentation of thinking structure could be a starting point to defragment their errors when constructing mathematics concepts and solve mathematics problems. In the construction of concepts, there are four kinds of defragmentation; scheme appearances, scheme knitting, logical thinking repair, and the repairment of analogical thinking. Meanwhile, in solving the problems, two defragmentations were found, namely connection appearances and scheme appearances. Herna (2016) focused on the students' defragmentation of pseudo-truth thinking in constructing the concept of limit functions. Five types of defragmentation were found; the establishment of holistic representations, the appearances of concepts, the appearances of connection, holistic thinking, and logical thinking's repairmen. The frameworks used by the two researchers were assimilation and accommodation. The current research focused on the emergence of the defragmentation of translation thinking structures based on CRA (checking, repairing, ascertain) framework (Wibawa, 2016). The students do checking by looking back at their answers, so they become aware of the errors. Repairing is how the students refine their answers on the basis of their awareness. Eventually, ascertaining is a process to assure that the revisions fulfill the expected answers to the problems. These steps are held to comprehensively understand the students' thinking structure's arrangement in solving mathematical modeling problems.

B. Methods

The current research employed a qualitative approach with a descriptive-explorative setting (Creswell, 2007). Eighty-five undergraduate mathematics students (4^{th} and 6^{th} semester) were involved in this research. We purposively chose the students since they had enrolled in calculus and learned the volume of the rotating objects in calculus. It means that the related concepts have been constructed either adequately stored or not in their schemes. A modeling problem was given to all students. To solve the problem, the students need to draw graphs and create mathematical models. The main concepts required are basic geometry and integrals.

Overall, procedures of data collection and analysis were carried out in three stages: categorizing data, reducing data, and drawing conclusions (Moleong, 2007) as follows.

Modeling problem

A company will produce a new gold necklace with two solid ball-shaped beads that look like the picture. The beads are formed by diametrically perforating them using a drill bit with a radius of 5 mm. For aesthetic purposes, the radius of a solid ball is determined to be 2 times the radius of the drill bit. The company wants to know the volumes of 2 beads are left on the necklace (before carving). Help this company to solve it!

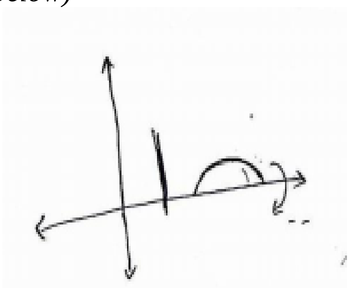
Firstly, we categorized all students' answers into two; correct and incorrect answers. The correct answers were not used in this research since it aimed to reveal and observe students' fragmentation of thinking structure. The incorrect answers were grouped into three sub-categories: (1) Students with very essential errors, where students did not realize that there were irregular shapes or curved residual spaces; (2) Students with essential errors, where students were aware of the irregular shapes or curved spaces but did not use definite integrals to solve them; (3) Students with less essential errors, where students were aware of the irregular shapes or curved spatial shapes and used definite integrals to solve it but the given answer was wrong. From the sub-categories, we purposively selected six prospective subjects (two students represent each sub-category) to be interviewed in a semi-structured format. Before that, the six students were given the same modeling problem. In re-answering the problem, the students were asked to think aloud. The interview process was recorded with one voice recorder and two cameras; one camera shot all students' bodies, and another camera focused on students' writing when answering the problem. The interview identified the students' errors and their causes, after which limited interventions to support students defragment their thinking were given. Three students were eliminated since they did not show any sign of defragmentation, while the other three then became the subjects of this research. The three subjects still represent each sub-category. **Secondly**, the subjects' works and interviews were further analyzed and reduced referring to correct understanding, incorrect understanding, the limited interventions (creating disequilibrium, scaffolding, and cognitive conflicts), the existence of fragmentation, CRA framework, and the defragmentation. The reduced data were displayed in figures (students' answers), diagrams (students' structure of thinking), and excerpts of students' interviews. **Lastly**, the findings were drawn in the form of a defragmentation pattern based on the CRA framework.

C. Findings and Discussion

The answers of eighty-five students show no correct answers: two unfinished answers and eighty-three complete answers. In this case, we focused on the complete answers. The complete answers were grouped into the three sub-categories. In sub-category 1 (very essential errors), there are sixty-three answers. Meanwhile, the number of answers in sub-category 2 (essential errors) and sub-category 3 (less essential errors) is seventeen and three, respectively. Three students representing each sub-category were selected. They underwent the fragmentation of translation thinking structure and are able to defragment their thinking. Each subject's defragmentation is unique. We found three types of defragmentation: defragmentation from verbal representations to graph representations, defragmentation from graph representations to symbolic (algebraic forms), and defragmentation from the graph and symbolic representations to mathematical models. The defragmentation of each subject is presented and elucidated as follows.

Subject 1 (S1)

S1 made a mistake in drawing on the Cartesian plane. The process of changing the representation from the drilled ball problem (verbal representation) to the graph in the Cartesian plane was not appropriate. Diagram 1 shows how S1 moved from incorrect understanding (Figure 1) to the correct one (Figure 2) after receiving the limited intervention.

Incorrect understanding	S1's statement: *(Drawing). If there is a line, then it is rotated using the ring method or anything else, a cylinder will be formed. If there is a curve like this (pointing to the figure below)*  **Figure 1**. S1's initial graph

Limited intervention	Facilitating disequilibrium: *Are you sure about the graph?* Providing scaffolding: *In this case, if it is viewed as a dimension of two spheres, then what will it be?*

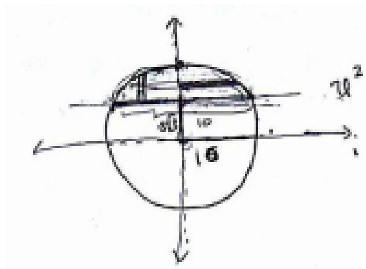
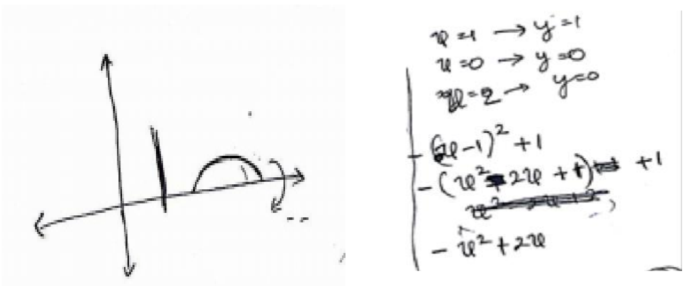
Correct understanding	*So, it is like this. If this (drawing…) can be like this. Obviously, this is not the diameter*  **Figure 2**. S1's revised graph

Diagram 1. S1' defragmentation from the verbal representation to the graph representation

As shown in Diagram 1, we provided a limited intervention in order to create a disequilibrium by asking his confidence in the constructed graph. Through the disequilibrium, S1 doubted his answer by paying attention to the information in the problem, such as radius and diameter (*checking*). Then we scaffolded S1 to trigger his sense of drawing on the Cartesian plane. S1 firstly drew a Cartesian diagram, then made a circle centered at (0,0) and drilled horizontally. Figure 2 shows the solid as the result of drilling in two dimensions. S1 was able to correct its initial understanding (*repairing*). He also ensured that the graph of the circle went through the center point (0,0) and the solid whose one of the curved surfaces is part of the circle (*ascertaining*). In terms of the structure of thinking patterns, S1 experienced fragmentation of translational thinking structure from verbal to graphics. Afterward, S1 defragmented his thinking after receiving the intervention from verbal to graphics.

Incorrect understanding	S1's statement: *...If it is like this (drawing ...), maybe use a quadratic function... since the form is just like that, meaning that the graph is a quadratic function that has a bounded domain...*  **Figure 3**. S1's initial equation

Limited intervention	Facilitating disequilibrium: *Are you sure about the equation? Just try it out!* Providing scaffolding: *What do you get if it is a part of the circle?*

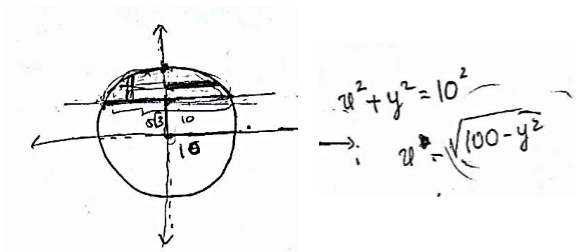
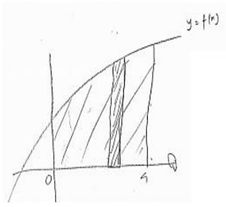
Correct understanding	S1's statement after correctly drawing the graph: *Oohhh... It can be determined using the circle formula; the function is a circle. Circle x square plus y square equal to r square.*  **Figure 4**. S1's revised equation

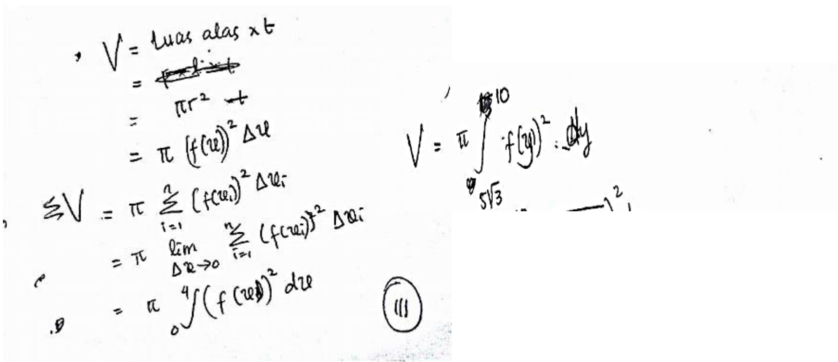
Diagram 2. S1' defragmentation from the graph to the algebraic form

Furthermore, S1 made an error in determining the equation of the graph in the Cartesian plane. Diagram 2 reveals S1's understanding and the given intervention. It shows that S1 understood the circle equation as a quadratic equation or quadratic function because the S1's graph looks like a parabola (Figure 3). Based on this error, we provided a limited intervention by facilitating S1 to experience disequilibrium through questions that could make him reconsider the initial answer. With the disequilibrium, S1 realized a mismatch between the graph and the equations (*checking*). After that, we gave the scaffolding, which makes S1 construct a new graph. Within the graph, S1 understood that the curved space is part of the circle, which means that the equation refers to the circle equation. In this case, S1 has corrected the error in determining the graph equation (*repairing*). S1 also ascertained the answer by stating that the graph on the Cartesian diagram with the center (0,0) and the curved space (in two dimensions) is part of the circle (*ascertaining*, Figure 4). Regarding the structure of thinking patterns, S1 experienced fragmentation of translational thinking structure from a graph to the algebraic form. Through the intervention, S1 defragmented his translational thinking by connecting the constructed graph to the circle as part of the drilled problem.

Lastly, S1 had difficulty connecting his understanding of integrals, slicing, rotating on one axis, the results of the rotation by finding a ring shape and the integral form. This made him unable to construct the integral form to determine the volume of the solid with one curved surface. Diagram 3 illustrates the changes in S1's understanding before and after receiving the intervention. We questioned S1's understanding to ensure that his thinking scheme was constructed entirely and to identify an unsystematic understanding. S1 restated his understanding of the concepts needed to apply the finite integral and realized the difficulties he experienced (*checking*). To support his understanding, we provided scaffolding (Figure 5) to create the scheme by asking him to do the slicing, but S1 did not determine the result of the slice correctly. We then encouraged S1 to simplify the problem into the area. He was able to determine the integral form, sliced, and approximated the number of slices. S1 seemed to have an understanding of the Riemann integral, which was applied to the area problem. We asked him to apply his understanding of Riemann integrals to the volume. S1 focused the calculation on the solid from the drilling results (not the one with one curved surface). He could improve his understanding of Riemann integrals' application to volumes by constructing the integral form correctly (*repairing*, Figure 6; Figure 7). S1 ensured his answers by stating that the integration boundary is determined through the intersection point on the y-axis and the function adjusts to the slices (*ascertaining*). From his structure of thinking patterns, S1 experienced a fragmentation of translational thinking with meaningless connections. Subsequently, S1 defragmented his thinking structure by linking previously unconnected concepts, starting from slicing and approximating the integral form (mathematical model).

Incorrect understanding	S1's statement: *...if it is rotated, maybe the volume can be determined. For example, if it is rotated toward the x-axis, then it will look like a ring, but depending on the axis to which it is rotated. I mean, taking a sample, by doing this or this way (Slicing...)* S1 incorrectly determined the boundary of the integral; the lower bound is 0, and the upper one is 5. He only paid attention to one slice made and considered the *x*-axis in determining the boundary.

Limited intervention	Scaffolding to create a scheme: *Please try now! Did you slice this way?* Simplifying the problem:  **Figure 5**. The illustration of the finite integral concept given to S1 as part of the intervention Scaffolding to connect the scheme: *Connect the problem you have* Creating disequilibrium: *Are you sure about this boundary?*

Correct understanding	S1' statement: About the area: *The area of the plane, rectangle, is its width multiplied by length, the length is f(x), and the width is delta x.* *If the total is added, it will be sigma. Sigma* $f(x_i)$ *delta* x_i*, i is from 1 to n. Ehh.. to indefinite.* About the volume:  **Figure 6**. S1's mathematical model S1 reflected, thought for a couple of minutes, and assured that the integral boundary was by *y*-axis, not *x*-axis. Then S1 determined the lower bound is root 3 and the upper bound is ten.

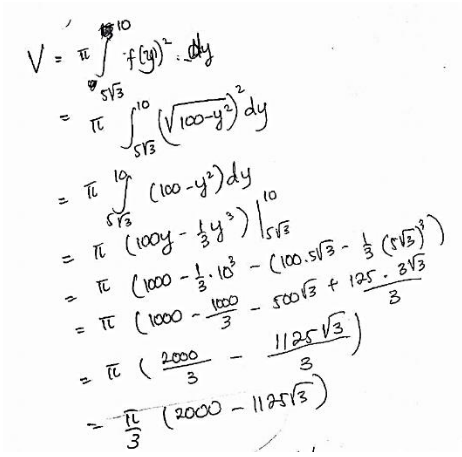
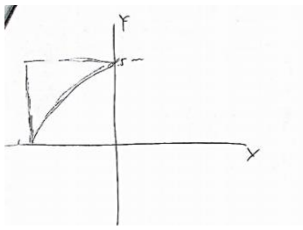
	 **Figure 7**. S1's calculation using the mathematical model

Diagram 3. S1's understanding before and after connecting the related concepts

Subject 2 (S2)

S2 made errors in changing the drilled ball's representation (verbal) to the graph on the Cartesian diagram. Diagram 4 shows S2's initial understanding, the given intervention, and her revised understanding. The graph made by S2 as the target representation was not complete, such as the sphere radius, the cylinder's radius, and the cylinder in the source representation (Figure 8). In this case, S2 experienced fragmentation of translational thinking from verbal to graphics. S2 defragmented her thinking by revising the graph (Figure 9). This process is called defragmentation of the structure of translational thinking from verbal to the graph. S2 considered the elements in the verbal representation (drilled ball) before drawing the graph. The defragmentation occurred after we facilitated disequilibrium and cognitive conflict.

Furthermore, S2 experienced difficulties connecting her schemes regarding Riemann integrals, functions, partitioning, and integral forms to determine the volume of a solid that has one curved surface. S2 thought that to determine the volume, a multi-variable integral concept or the Riemann integral can be used. However, the concept could not be constructed properly, so she was difficult to determine it. Diagram 4 shows the changes in S2's understanding.

Incorrect understanding	S2's statement: *I use this way to the graph in two dimensions (Drawing…). Since it is the circle here, this is to determine the smallest h, just one-eight.*  **Figure 8**. S2's initial graph

Limited intervention	Facilitating disequilibrium: *Is the graph correct?* Facilitating cognitive conflict: *This is obvious, the center is clear. What about that?*

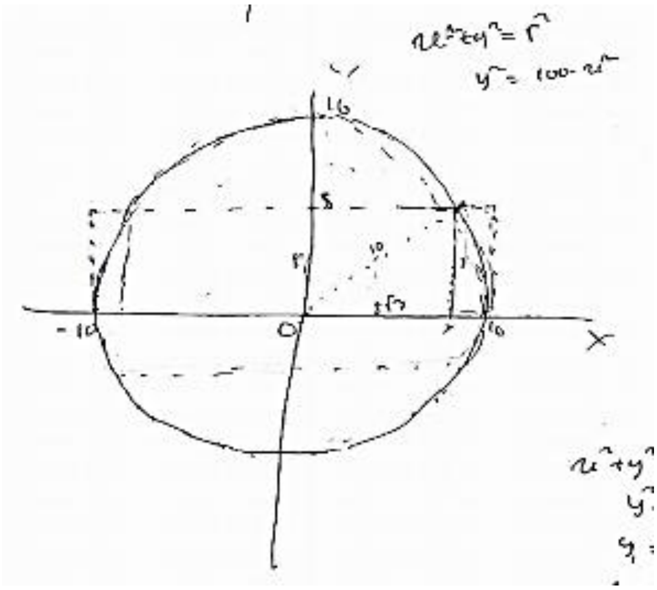
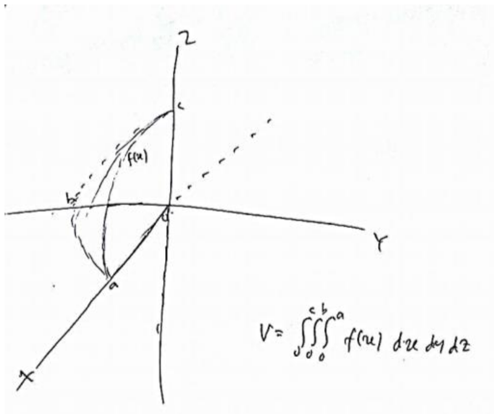
Correct understanding	S2' statement: *If I make another graph (drawing…). This is x and y; then the drilling is 10 and 5. To calculate, it is 10*  **Figure 9**. S2's revised graph

Diagram 4. S2's understanding before and after defragmenting her structure of translational thinking from verbal to graph

Diagram 5 shows that S2 was not able to construct both procedurally (the sequence of completion) and conceptually (how and why these concepts are used). In this case, S2 experienced meaningless connections, which means that the schemes presented are not fully understood so that they cannot be used to solve the problem (Figure 10). S2 defragmented her thinking by linking the schemes she has through the given scaffolding; Asking S2 to simplify the problem as the application of the Riemann integral to determine area and associate it with volume. S2 re-arranged her thinking structure starting from slicing, approximating, and forming mathematical models or definite integral forms (Figure 11).

Incorrect understanding	S2's statement: *In the multiple variable calculus, if it is to rotate, assume a circle (drawing…); is it a sphere? It has a function for the volume, assume f(x), so the volume is this integral (writing the integral form…).*

	 **Figure 10**. S2's initial graph and integral form for the volume

Limited intervention	Simplifying the problem by determining the area Providing scaffolding to connect to the volume

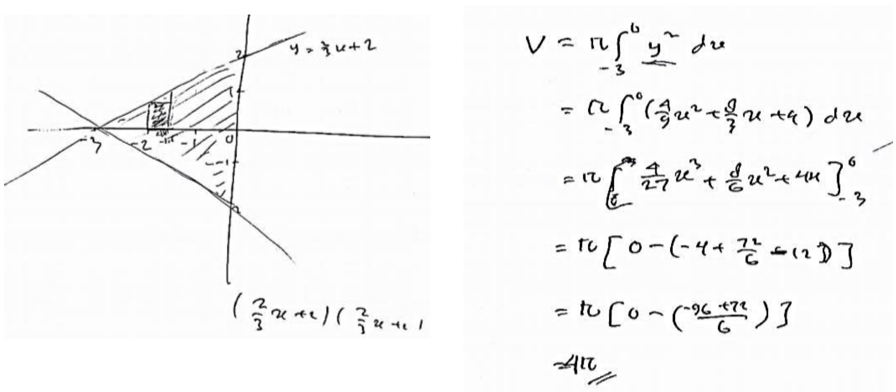
Correct understanding	S2's answer: S2 constructed the area of integral to apply the Reiman integral on the area  **Figure 11**. S2's revised graph and volume calculation The area: Slicing, approximating, and forming integral forms. The volume: Determining upper and lower bound, a function using the circle equation by changing it to $y = \sqrt{100 - x^2}$ and the integral form.

Diagram 5. S2's initial understanding and after defragmenting her thinking by connecting the concepts

S2 was able to connect her existing scheme to apply the Riemann integral to the volume of rotating objects, but she still had the error in connecting the integral form with the area of integral. Diagram 6 illustrates changes in S2's understanding. The change of the representation from the graph to the mathematical model made by S2 was not fully correct. She did not pay attention to her constructed graph, such as the graph on the area of integral and the graph on the circular equation. This caused her incorrectly determine the volume using the integral form. S2 reconsidered the graph after receiving the limited intervention through the creation of cognitive conflict. In terms of the structure of thinking, S2 experienced fragmentation of translational thinking structure from graphs to models (integral form). Then she was able to defragment her thinking structure by translating back from the graphs and the algebraic forms to the models.

Incorrect understanding	S2's statement: *So, to evaluate the volume, if I use the same method as the previous one, phi is this. The interval is from null to root 3, the function is x, meaning that x square minus one.* The integral form $\int_0^{5\sqrt{3}} y^2 dx$ in S2's perception is to determine the remaining area of the drilled sphere. In fact, the form includes the cylinder.

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Limited intervention	Facilitating a cognitive conflict: *This is y square because we want to calculate the cylinder. What about that?*

⇩

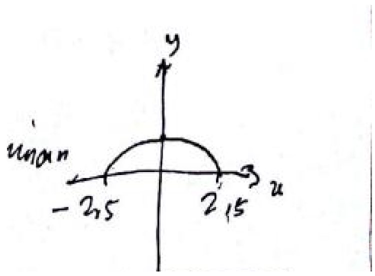
Correct understanding	S2's statement: *Oh… it includes this area as well. (Thinking in some minutes…) Ohh, yes, if the r is here, so it starts from here, this is as well (pointing to the graph). I just subtract this volume with this one.* S2 determined the volume of the cylinder by subtracting with the results of $\int_0^{5\sqrt{3}} y^2 dx$

Diagram 6. S2's initial understanding and after defragmenting her structure of translational thinking

Subject 3 (S3)

S3 made an error when changing the representation from a solid with one curved surface (source representation) to the graph (target representation) since she did not pay attention to the elements of the solid. In this case, S3 underwent a fragmentation of translational thinking structure from verbal to graphics. She was capable of re-structuring her thinking structures by re-translating; Considering the elements in the solid. It is called the defragmentation of the thinking structure between schemas in the source representation and the target representation. Diagram 7 illustrates S3's initial understanding and the changes in her understanding after receiving the scaffolding.

The diagram shows the checking; S3 observed the graph (Figure 12) and noticed an error. This awareness was obtained when S3 tested the equations, where the results did not fit the equations. We then gave the scaffolding. Through this scaffolding, S3 reconstructed the graph (Figure 13), with information that the curved (in two dimensions) solid is part of a circle. She drew a circle centered at (0,0) and made a drilling line in the direction of the y axis. She corrected the error by constructing the correct graph or source and determining the target representation (*repairing*). Subsequently, she ensured that the curved part is part of the circle and shades it (*ascertaining*).

Incorrect understanding	Drawing on the Cartesian plane by positioning the midpoint of the remaining solid, which has the curved surface at the center point (0,0).  **Figure 12**. S3's initial graph

Limited intervention	Providing scaffolding: *Can you relate it to the existing graph?*

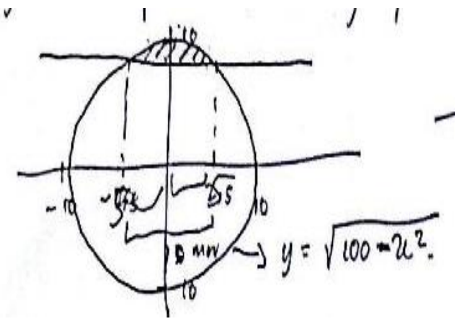
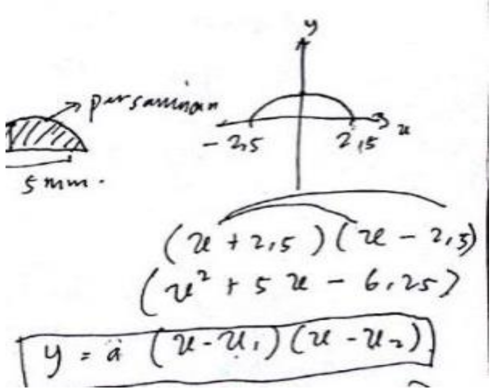
Correct understanding	*This… (Pointing to the graph) which I previously drew outside the Cartesian plane. Is this Sir? (Pointing to the remaining part of the solid which has the curved surface)*  **Figure 13**. S3's revised graph

Diagram 7. S3's initial understanding and after defragmenting her translational thinking structure from verbal to the graph

S3 also incorrectly determined the equation of the sphere where one of its surfaces is curved (Diagram 8). S3's error was caused by incorrect translation from graphs to the algebraic forms and her assumption about a quadratic function (Figure 14). In this case, S3 experienced fragmentation of translational thinking structure from graphs (source representations) to algebraic forms (target representations). S3 defragmented her thinking structures by re-translating; Considering the elements of the graph she made, correctly constructing a new graph, and connecting the previous graph which is a drilled ball with the graph on the Cartesian plane (Figure 15). She noticed that the part of the graph created is part of a circle, so the equation is $x^2 + y^2 = 100$. The defragmentation from the graph to the algebraic forms is facilitated through the provision of limited interventions. In the interview excerpt below (Diagram 8), S3 initially misunderstood the problem, and then S3 made corrections. The way to a correct understanding is the defragmentation of the structure of translational thinking from the graph to the algebraic form, involving checking, repairing, and ascertaining.

Incorrect understanding	*Because it looks like a bowl (pointing to the graph). The graph is more or less like this (hand gesture in the form of a parabola) with the intersection of the x-axis here and the y-axis here (pointing to the graph). But is this the maximum point?*  **Figure 14**. S3's initial graph and equation *Because it is curved like this, then it is a quadratic function*

Limited intervention	Facilitating disequilibrium: *Are you sure about a quadratic function?* Providing a scaffolding: *You can relate it to your constructed graph*

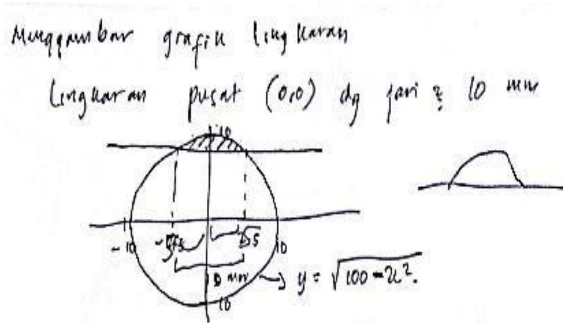
Correct understanding	*Oya, it is a circle, because of this (bold her constructed graph) then cut like this, from here to here (pointing to the graph)*  **Figure 15**. S3's revised graph and equation

Diagram 8. S3's initial understanding and after defragmenting her translational thinking structure from graph to algebraic form

S3 did a checking by noticing the area of the integral (a solid as a result of drilling) and observing the boundaries of the integral, at which she realized that her initial boundaries were incorrect due to the horizontal slicing. Then she determined the boundaries using the Pythagorean theorem. She was able to determine the boundaries correctly and made appropriate integral form (*repairing*). S3 made sure that the integral form is correct with a strong argumentation (*ascertaining*). Diagram 9 shows S3's defragmentation of translational thinking structure from the graphic and algebraic form to the mathematical model after receiving the interventions.

Incorrect understanding	S3 determined the volume of the sphere using the following integral form $$Vsphere = \pi \int_0^{10} \left(\sqrt{100 - x^2}\right)^2 dx$$ S3 determined the volume of missing parts of the sphere $$Vsphere = \int_0^{2,5} \left(\sqrt{25 - x^2}\right)^2 dx$$

Limited intervention	Facilitating disequilibrium: *Why is the mathematical model like this?* Providing scaffolding: *Please try slicing!*

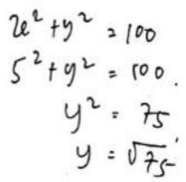
Correct understanding	*Like this Sir, horizontal (thinking in some minutes…). Oya, the boundary starts from this bottom to the top Sir (pointing to the graph). But this is still unknown? (thinking in some minutes). Oya, I can use the Pythagorean theorem*  S3 determined the integral form of the volume of the drilling space $$V = \pi \int_{-\sqrt{75}}^{75} -y^2 + 75\, dy$$

Diagram 9. S3's initial understanding and defragmentation of translational thinking structure from graphics and algebraic forms to mathematical models

Defragmentation of translational thinking structure from the verbal to the graph

To determine the students' defragmentation of translational thinking structure from verbal to graphic, we conducted in-depth interviews so that they were able to express what was on their minds. In other words, we attempted to reveal how the students understand all the important elements in the problem and used to construct a graph to solve the problem. Diagram 10 illustrates how the students defragment their structure of translational thinking from verbal to graphical based on the CRA framework.

When students deal with word problems that require mathematical modeling, they tried to understand the facts on the problem and constructed graphs which help them solve the problem. Students constructed the graphs by utilizing their previous incomplete understanding of the facts. This resulted in errors in constructing the target representation. The errors did not make students revise their work because of limited awareness. For this reason, we provided limited interventions that aim to make students aware of the errors and improve the works using their understanding. We facilitated students to experience disequilibrium by questioning their works on the problem. Furthermore, cognitive conflicts were supported so that students reflect on their answers. After the interventions, students were able to realize the errors by noticing all the facts on the problem (*checking*). Then students reconstructed the target representation based on their new understanding. In constructing the target representation in the form of a graph, students paid attention to all the facts on the problem. After correctly constructing the graph, they have carried out the *repairing* process. To ensure that their understanding was obtained through high awareness, we asked students to explain the construction of the new graph again. The students were said to be doing *ascertaining* if they were able to explain the process of constructing the graph correctly.

Defragmentation of translational thinking structure from the graph to the symbol or algebraic form

We administered depth interviews in order to examine students' defragmentation of translational thinking structure from the graph to symbol or algebraic form. It aimed to reveal how students grasp essential elements in a graph and make an equation that can help solve the problem. Diagram 11 illustrates the students' defragmentation referring to the CRA.

When students solve mathematical modeling problems, they must be able to change various forms of representation, including graphical representations into algebraic forms. In this study, students constructed equations by referring to the graphs they made. In fact, the graphs hinder students' understanding of the correct equations, resulting in incorrect answers. Students who were supposed to determine the circle equation, on the contrary, determine the quadratic equation. The discrepancy between the source representation in the form of a graph and the target representation in the form of an equation in algebraic form is indicated by the red circle contained in each scheme (Diagram 11). Then we gave limited interventions to students. Students are aware of the errors; for example, the construction of the graph was inappropriate because it did not comply with all the facts contained in the problem. They reconstructed the graph using their new understanding. Based on the graph, students could easily construct the circle equation. This shows that the *repairing* process was successfully carried out by students. We then asked the students to explain again how the process of a quadratic equation can turn into the circle equation. They were able to explain correctly in accordance with applicable mathematical rules.

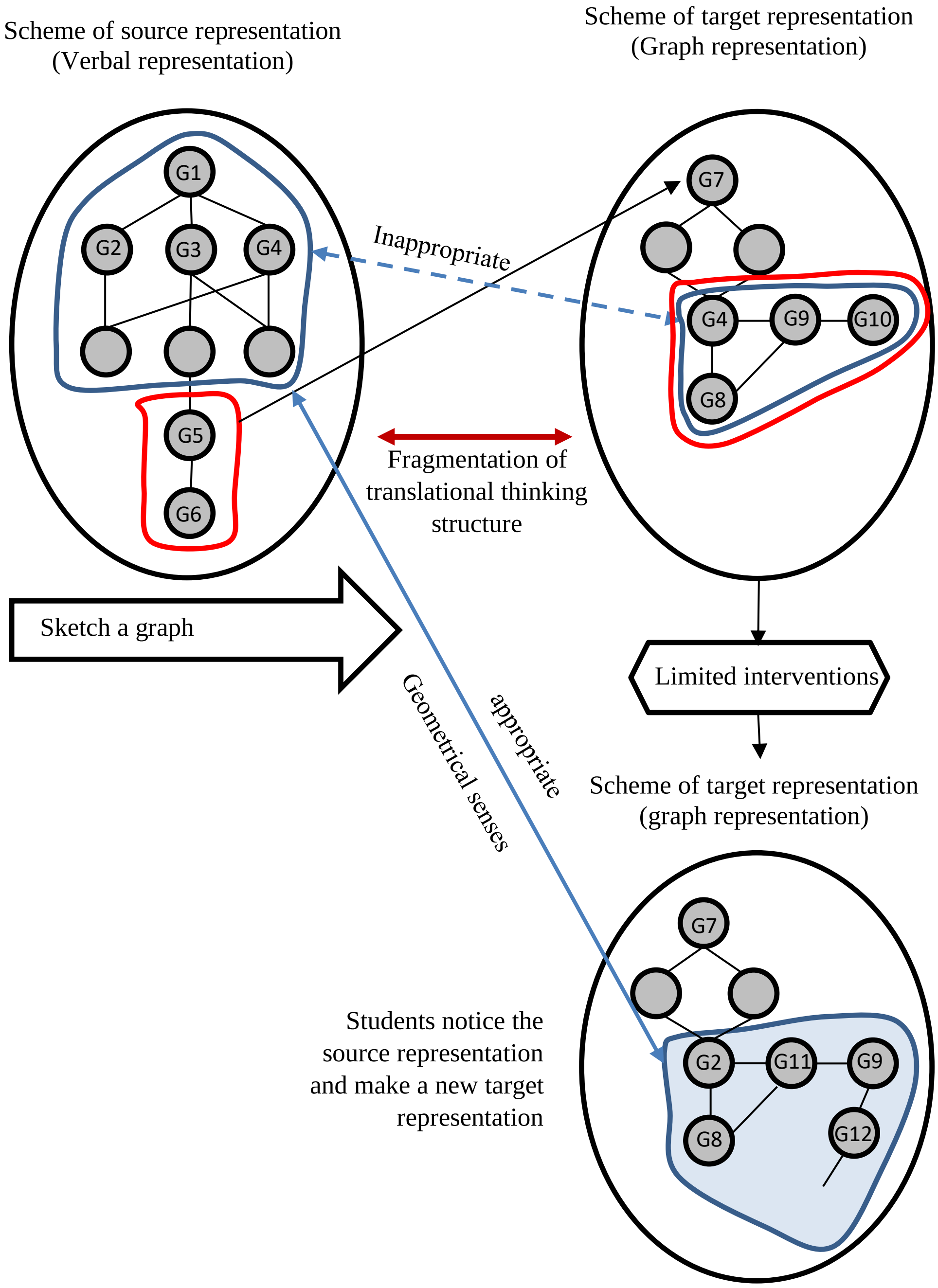
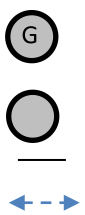


Diagram 10. The illustration of defragmentation of translational thinking structure from verbal to graph



Concepts or ideas constructed by the students

Sub-concepts or ideas constructed by the students

Steps of problem-solving which connect schemes (connector of the constructed schemes)

The connections of thinking structures on the source representation and the target representation which is not appropriate

↔ The connections of thinking structures on the source representation and the target representation which is appropriate

↔ The connections of thinking structures on the source representation and the target representation which show the fragmentation of translational thinking

Thinking structures that have errors

limited interventions

thinking structures that have been constructed

thinking structures that have been defragmented and are in line with the problem structure on the source representation

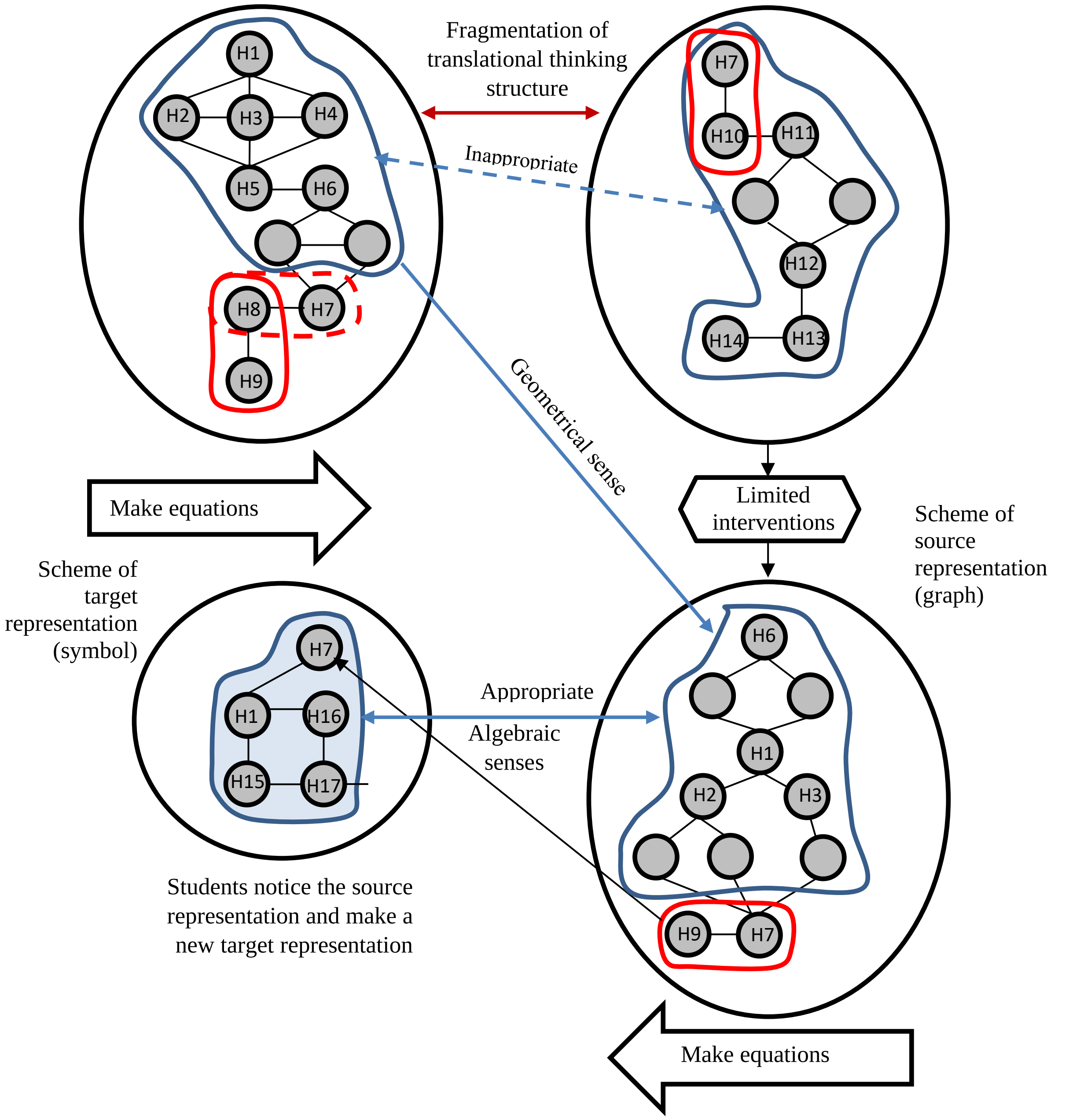


Diagram 11. The illustration of students' defragmentation of translational thinking structure from the graph to the symbol or algebraic form

Defragmentation of translational thinking structure from the graph and symbolic (algebraic form) to the mathematical model

The students were expected to find the correct mathematical model so that it can be used to solve the problem. Through in-depth interviews, we reveal how students defragment their translational thinking structure from graph and symbol (algebraic form) to mathematical models. More specifically, how students understand important elements in graphs and equations linking to their existing concepts to make mathematical models that help solve the problem. Diagram 12 illustrates the students' defragmentation based on the CRA framework.

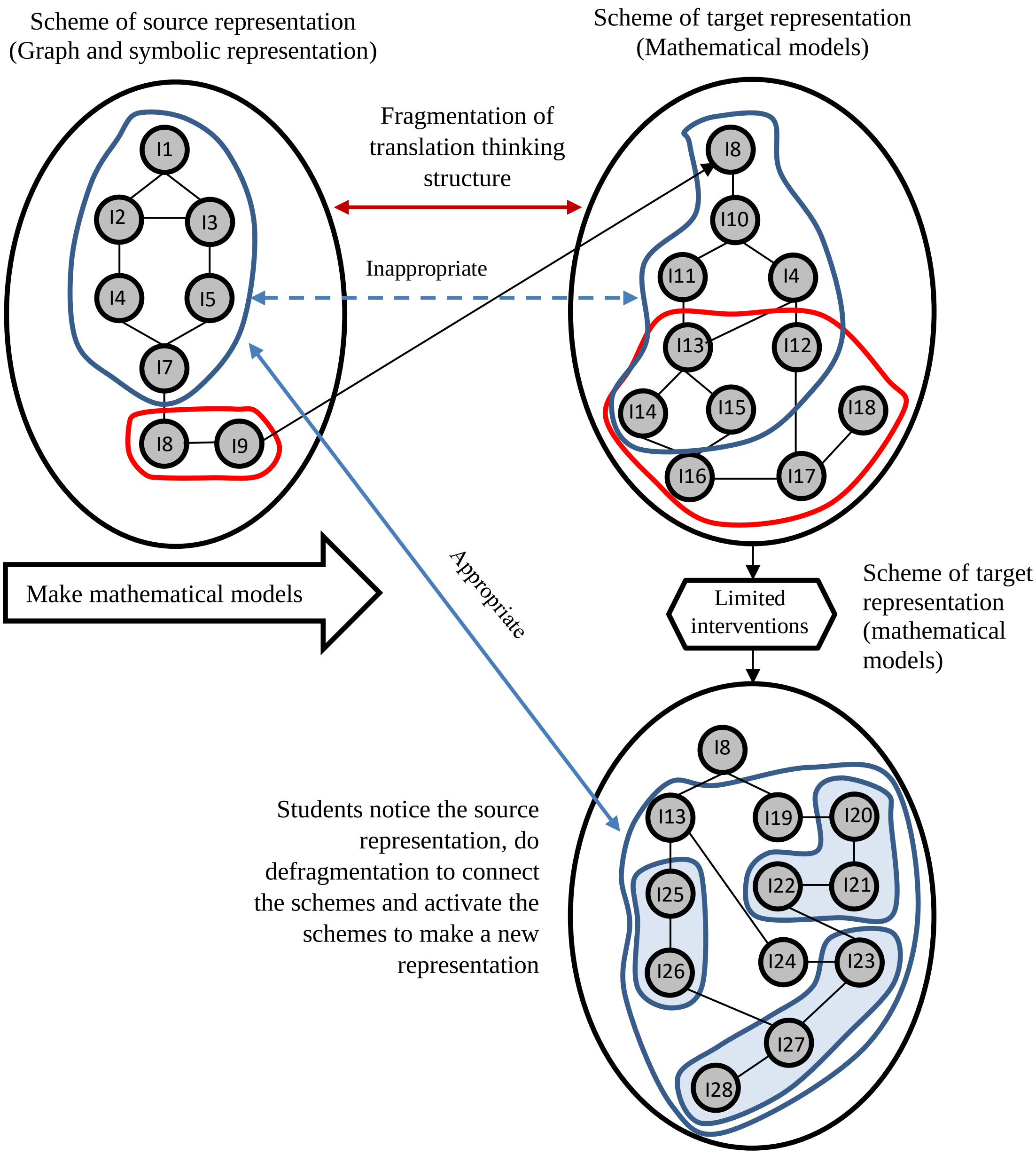


Diagram 12. The illustration of defragmentation of translational thinking structure from graphs and symbols to mathematical models

To construct a mathematical model from the given problem, students need various concepts that must be recalled. In addition, they also need to consider the facts on the problem. Since the level of construction of this mathematical model is very complex, students experienced difficulties and make various errors. It is due to the incomplete understanding of what they have learned before. Incomplete understanding that results in errors does not mean that students forget everything they learn and are unable to improve (Subanji, 2015). The blue area in Diagram 12 shows how students construct mathematical models by reconnecting previously incomplete concepts. Students restructure their schemes through the limited intervention to complete their understanding and then reconnect it with other concepts. In this case, students' success in constructing new mathematical models is based on their own efforts to recall and relate concepts that are appropriate to the problem. Subanji (2016) calls this situation as the students' attempt to connect the schemes, which were previously not well connected.

D. Conclusion

Regarding the importance of supporting students in mathematics problem-solving, especially how to defragment their translational thinking structure, this study offers insightful findings to do that through checking, repairing, and ascertaining (CRA) with some limited interventions such as scaffolding or cognitive conflicts. The three processes: looking back at their answers, so they become aware of the errors, refining their answers on the basis of their awareness, and assuring that the refinement solves the problem, do help students' defragmentation. The current study found three types of defragmentation when solving a mathematical word problem that requires modeling; defragmentation from verbal representations to graph representations, from graph representations to symbolic representations (algebraic form), and from the graph and symbol to mathematical models. The students who are difficult or have errors in solving mathematical problems cannot be viewed as their drawbacks in learning mathematics. However, the difficulties or errors also relate to storing and calling back their existing knowledge, which leads to pseudo-knowledge. For this reason, it is vital to provide the students more opportunities to restructure or defragment their thinking when experiencing difficulties or errors in mathematical problem-solving.

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